How to Fall into a Black Hole

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Ikepod Horizon Black Hole RS (IHHB10)

Can a finite temperature CFT describe an object falling through the horizon of an AdS black hole?
Figure 1: The firewall argument in a nutshell: quanta $r$ and $e$ can escape the black hole thanks to entanglement with Hawking partners $b$ and $a$. Monogamy prevents the partners to get entangled with other interior quanta. So after the Page time, the black hole is in a maximally mixed state, and $a$ and $b$ can not escape via the normal Hawking process.

$$|0_U\rangle = \sum_n \sqrt{w_n} |n\rangle_a |n\rangle_b.$$
Figure 2: When the Hawking partner $b$ leaves the black hole, it can not be entangled with any mode $a$ right behind the horizon: the horizon turns into a firewall. To avoid this conclusion, one needs a mechanism for allowing entanglement to re-establish inside the black hole.
“What are the characteristics of a black hole?”
A black hole is always in a mixed state!

\[ |i\rangle \in \mathcal{H}_M \quad N = \dim \mathcal{H}_M = e^{S_{BH}}, \quad S_{BH} = 4\pi M^2. \]

\[ |\Psi\rangle = \sum_{\tilde{i} \in \mathcal{H}_{\text{code}}} a_{\tilde{i}} |\tilde{i}\rangle |\Phi_{\tilde{i}}\rangle \quad \leftrightarrow \quad \rho_{BH} = \sum_{\tilde{i} \in \mathcal{H}_{\text{code}}} p_{\tilde{i}} |\tilde{i}\rangle \langle \tilde{i}| \]
Hilbert space splits up into three sectors:

\[ \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_R^{\text{early}} \]

with

\[ \mathcal{H}_A = \mathcal{H}_{BH}, \quad \mathcal{H}_B = \mathcal{H}_R^{\text{late}}. \]

\[ H = H_{BH} + H_R + H_{\text{int}}, \]

Hamiltonian evolution in Interaction Picture
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Hamiltonian evolution in Interaction Picture
Initial mixed state

\[ \rho_{AB}(0) = \sum_{\tilde{i}} p_{\tilde{i}} |\tilde{i}\rangle \langle \tilde{i}| \otimes |0\rangle_b \langle 0|, \]

 evolves into

\[ \rho_{AB}(\tau) = U \rho(0) U^\dagger = \sum_{n,m} \rho_{nm} |n\rangle_b \langle m| \]

\[ \rho_B = \sum_n w_n |n\rangle_b \langle n| \quad ; \quad w_n = \frac{e^{-\beta E_n}}{Z} \quad ; \quad S_B = \log Z + \beta \bar{E} \]

B = Hawking atmosphere
Time evolution in terms of Kraus Operators \( C \)

\[
U(\tau) |i\rangle |0\rangle_b = \sum_n C_n |i\rangle |n\rangle_b.
\]

Unitary:

\[
\sum_n C_n^\dagger C_n = 1_M.
\]

**"BAD"**

\[
C_n C_m^\dagger = \frac{1}{Z} \delta_{nm} 1_{M-E_n}
\]

**"GOOD"**

\[
\langle \bar{i} | C_n^\dagger C_m | \bar{k} \rangle = w_n \delta_{nm} \delta_{\bar{i}\bar{k}}.
\]

C’s are ergodic `random' matrices with known statistical properties!
We can compute the Page curve of our model!

\[
\rho(E)^2 = \left( \frac{w_E}{N_c} + 2 \frac{d_B(E)}{N Z} \right) \rho(E) - \left( \frac{d_B(E)}{N Z} \right)^2 1_{M-E}
\]

\[
S(E) = w_E d_B(E) \left[ \log \left( \frac{N Z}{d_B(E)} \right) - \sqrt{\frac{y}{2 + y}} \log \left( 1 + y + \sqrt{2y + y^2} \right) \right]
\]

\[
y = \frac{N}{N_c 2d_B(E)} e^{-\beta E},
\]

**Two limiting cases:**

**young BH:**
\[
S_{AB} = \log N_c, \quad S_A = \log N_c + \log Z + \beta \bar{E}
\]

**old BH:**
\[
S_{AB} = \log N, \quad S_A = \log N - \beta \bar{E}
\]
Key Tool: Recovery Super Operator $R$

\[ R_n = \frac{1}{\sqrt{W_n}} \sum_{\tilde{i}} \tilde{i} \langle \tilde{i} | C_n^\dagger, \]

\[ R | j \rangle | 0 \rangle_a = \sum_n R_n | j \rangle | n \rangle_a \]

\[ RU | i \rangle | 0 \rangle_a | 0 \rangle_b = \begin{cases} | i \rangle | 0_U \rangle & \text{if } | i \rangle \in \mathcal{H}_{\text{code}} \\ 0 & \text{if } | i \rangle \notin \mathcal{H}_{\text{code}} \end{cases} \]

\[ | 0_U \rangle = \sum_n \sqrt{W_n} | n \rangle_a | n \rangle_b. \]

Produces the Unruhu Vacuum!
Reconstruction of Interior Operators:

\[ \phi(y) = \langle 0 | R^\dagger \phi(y) R | 0 \rangle_a. \]

A acts on black hole Hilbert space satisfies QFT operator algebra.
Dpbranesq In this comparisono the AdS space replaces the black hole interior geometryo the matches the number of infalling partner modes of the Hawking pairsq This suggests that the outgoing field $V$

$\text{tr} (\rho \phi_a(y) \phi_b(x)) = \langle 0_U| \phi_a(y) \phi_b(x) | 0_U \rangle$, \text{horizon}.

References
Error of the Recovery Operation:

\[ \Pi U |\bar{i}\rangle|0\rangle_b = U |\bar{i}\rangle|0\rangle_b + \text{error}. \]

\[ \text{error} = EU |\bar{i}\rangle|0\rangle_b, \quad \text{with} \quad E = \sum_n \frac{N_{\text{code}}}{N_n} 1_{M-E_n}. \]

Reconstruction breaks down for maximally mixed states!

Firewall Paradox

AMPS
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Reconstruction works for all states $|\psi\rangle$ that fit inside a code space!

Firewall Paradox? AMPS
Decoherence

In practice this means that not all of the quantum information in the early state can be extracted in real time, since it is unsuitable for delicate quantum measurements involving interference experiments, etc. In other words, it is unavoidable that part of the information contained decoheres and becomes, as far as the infalling observer is concerned, classical information. An essential difference between classical and quantum information is that the latter is delicate and exclusive – it cannot be cloned, and while it can be highly non-local, it cannot be simultaneously accessed at different locations first can be duplicated – once it is known in some location it can be broadcast throughout space and become knowledge that is shared by different semi-classical observers.

\[ \mathcal{O}_{BH} = \sum_{C} \mathcal{O}_{BH}^{(C)} \otimes \left| \Omega_{C} \right\rangle \left\langle \Omega_{C} \right| \]

Does this preserve linearity and locality?
Decoherence

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\[ O_{BH} = \sum_{c} O^{(c)}_{BH} \otimes |\Omega_c\rangle\langle\Omega_c| \]

Solving the problem:
Does this preserve linearity and locality?
Some conclusions:

* Using QECC, we’ve given a general construction of local QFT operators relative to which the horizon is smooth.

* Generic BH states describe a semi-classical space-time with an interior geometry and a smooth event horizon.

* Maximally mixed BH state = sum of semi-classical states Einstein locality + general logic => old BHs have no firewall

Some outstanding challenges:

Formulate of the rules of observer complementarity

Trace the flow of quantum information and entanglement

Quantify violations of locality and limitations of effective QFT.
Code bits entangles qubits with environment, produces classical information.
Code bits assists QT protocol

Measurement:
enables the quantum teleportation of Hawking quanta back into the black hole interior
Space-Time Complementarity:

Space-time complementarity: A variable cut-off scale $\epsilon(x)$ on a Cauchy surface $\Sigma$ provides a permissible semi-classical description of the second quantized Hilbert space, only when the quantum fluctuations of the local background geometry induced by the corresponding stress-energy fluctuations do not exceed the cut-off scale itself. All critical cut-off scales that saturate this requirement provide complete, complementary descriptions of the Hilbert space.

Kiem, EV & HV, 1995
Fig 1b. This figure shows two space-like separated observables such as the infalling and outside observer on a black hole background! The consequences of this assumption are particularly striking in a situation with a center of mass energy that grows as \( e^{\Delta t/4M} \). The proposed complementarity principle states that observables for which this collision energy exceeds some (possibly macroscopic) critical value do not simultaneously exist as mutually commutable operators, however, we are inclined to use very different bases is assumed to be the natural setting for considering the simultaneous measurement by these two observables.

We then discover that the past history of these observables is fundamental in these observables, since the two observables are space-like separated. The field modes associated with these observables have a different cut-off in and near or just behind the horizon, of typical wavelength \( \lambda \). The field modes associated with these observables have a different cut-off in and near or just behind the horizon, of typical wavelength \( \lambda \). To illustrate this, let us consider the simultaneous measurement of an outside and a null radioactive source. The decay rate of an outside radioactive source is exponentially decaying as \( e^{-\lambda t} \), and thus very different from that of a null radioactive source. However, the free field propagation of the modes containing the modes of these two observables must consider their past history. In first instance, we can try to accurately compute transition amplitudes involving the modes of the observables at different times.

\[
E_{\text{cm}} \sim \frac{e^{\Delta t/4M}}{\lambda_{\text{in}} \lambda_{\text{out}}}
\]