Entanglement as the fabric of spacetime/quantum matter

Brian Swingle
Caneel Bay, Feb. 7
Acknowledgements

• Collaborators: Liza Huijse, Subir Sachdev, Senthil, Jeremy McMinis, Norm Tubman, John McGreevy
Quantum matter unified

Gravity and strings

Quantum information

Solid state physics

ENTANGLEMENT

Quantum engineering
Why entanglement?

How to think about $|\psi\rangle$?

$2^N$ complex numbers (N spins) is not applicable.

Quantum circuit that makes $|\psi\rangle$ from a product state.

$BQP \subset PSPACE$
Entanglement

\[ \rho_A = \text{tr}_B(\rho_{AB}) \]

Rényi entropy:

\[ S_n = \frac{1}{1-n} \ln(\text{tr}(\rho_A^n)) \]

\[ S_n(A) = S_n(B) = \ln 2 \]
Boundary "law"

\[ S(A) \sim L^{d-1} \]

\[ L \]

\[ \text{B} \]
Measuring entanglement

We can measure* $S_n, \ n > 1$

(particularly relevant $\rightarrow$ QMC Kallin-Isakov-Inglis-Melko-Hastings-...)

* May require a quantum computer. Quantum computer not included. Exponentially small signal.
Example: Fermi surface

\[ S = \int d^2 x dt \left[ \psi^* i \partial_t \psi - \frac{1}{2m} \nabla \psi^* \nabla \psi + \mu \psi^* \psi \right] + \int d^2 x \, d^2 y \, dt \, \psi^* \psi(x) U(x - y) \psi^* \psi(y) \]

- Entanglement entropy, shape dependence
- Renyi entropy
- Finite T generalization
- Multiple regions, non-convex regions
- Mutual information, …
Formulas

\[ S_n(A) = \left(1 + \frac{1}{n}\right) \frac{1}{24} \frac{1}{(2\pi)^{d-1}} \int_{\partial A} \int_{\partial \Gamma} |n_x \cdot n_k| \ln \left( L_{\text{eff}} \right) \]

(Klich-Gioev '06, BGS '09)

\[ \delta S_n = f_n \frac{1}{4} \int_{\partial A} \int_{\partial \Gamma} |n_x \cdot n_k| \frac{\cos \left( 2k_F L_{\text{eff}}(x,k) \right)}{(2k_F L_{\text{eff}}(x,k))^2/n} \]

(BGS-McMinis-Tubman '12)

\[ S(A,T) = \frac{1}{12} \frac{1}{(2\pi)^{d-1}} \int_{\partial A} \int_{FS} |n_x \cdot n_k| \ln \left( \frac{v_F}{\pi T \epsilon} \sinh \left( \frac{\pi T L_{\text{eff}}(x,k)}{v_F} \right) \right) \]

(BGS '10)

Entropy of a disk, radius L, T=0

\[ S_n(L) = a_1 k_F L \ln (L) + a_2 L + a_3 (k_F L)^{-a_4} \cos (a_5 k_F L) \]
Fermi gas (U=0)

A = disk, radius L

(BGS-McMinis-Tubman '12)
Fermi liquid

\[ A = \text{disk, radius } L \]

(McMinis-Tubman '12)
Proving universality

- “entanglement sum rule” for some models of fermions (f) coupled to other stuff

\[ S(L, T) = S_f(L, T) + S_\tau(L, T) \]

(BGS ’12, Yao-Qi ’10, BGS-Senthil ’11)

- Entropy is exactly additive, valid at finite T, all Renyi entropies, all parameters
- App. 1: Widom formula in a Fermi liquid
- App. 2: Thermal-entanglement crossover is universal
So what?

• We’ve crushed the Fermi liquid problem

• Combined with many other examples (e.g. gapped and conformal quantum matter), we begin to understand the detailed RG structure of entanglement

• ... so let’s use this knowledge to say something useful about quantum matter and holography
To (partially) “solve” condensed matter physics we must have a class of wavefunctions that can represent all interesting states and can be efficiently manipulated.

To (partially) “derive” holography we must show how the gravitational spacetime and locality emerge from appropriate types of quantum matter.
**Multiscale Entanglement Renormalization Ansatz**

- State built from tensor network; combines entanglement and coarse-graining (Vidal ’07)

Spin chain

$$\begin{align*}
&\downarrow \quad \downarrow \\
\text{State that encodes "entanglement per scale"}
\end{align*}$$

$$U_{RG,2\rightarrow1} \langle \psi_{UV} \rangle = \langle \psi_{ren} \rangle$$

= disentangler

= coarse graining
Entanglement Renormalization

- Procedure: keep renormalizing state until no entanglement remains

\[ UV: r = \epsilon, \ IR: r \rightarrow \infty \]

\[ k \text{ sites} \rightarrow 1 \text{ site}, \ r = \epsilon k^u \]

\[ |\psi_{IR}\rangle = U_{RG}(u_{IR})...U_{RG}(1)U_{RG}(0)|\psi_{UV}\rangle \]

- Gapped \[ u_{IR} \sim \log_k \left( \frac{\xi}{\epsilon} \right) \]
- Gapless \[ u_{IR} \sim \log_k \left( V^{1/d}/\epsilon \right) \]

\[ \xi \text{ correlation length, } V \text{ volume, } d \text{ space dim.} \]
Emergent discrete space

Entanglement of a UV region is controlled by the minimal number of bonds that must be cut to isolate it.
\[ \mathcal{I}(A, B) = S(A) + S(B) - S(AB) \]  

(BGS '12)
Correlation Functions

RG direction
Emergent holographic space

1. Emergent direction associated with RG flow
2. Quasi-geometric formula for entanglement entropy
3. Quasi-geometric formulas for various n-point functions
4. Local at the level of the graph
(a possible) Continuum version  

Pros:
- Feels closer to a continuum theory
- Can obtain some formal results
- Complete for free fields

Cons:
- Much harder to compute with
- Not clearly the right extension

\[ |\psi_{UV}\rangle \rightarrow |\psi_{IR}\rangle \]  

Verstraete et al.
RG circuit

\[ U_{RG} \psi_{UV} = U_{RG}^{-1} \psi_{IR} \]

Example for the rest of the talk:

\[ U_{RG} = e^{iDu_{IR}} \]

1. UV is regulated CFT ground state
2. D is a regulated dilatation operator
3. Space dimension \( d = 1 \)
A check: correlations

Primary field $\mathcal{O}(x)$, dimension $\Delta$

$$e^{iuD} \mathcal{O}(x) e^{-iuD} = e^{-u\Delta} \mathcal{O}(e^{-u}x)$$

$$\langle \psi_{UV} | \mathcal{O}(x) \mathcal{O}(0) | \psi_{UV} \rangle = ?$$

$$= \langle \psi_{IR} | e^{iu_{IR}D} \mathcal{O}(x) e^{-iu^*D} e^{iu^*D} \mathcal{O}(0) e^{-iu_{IR}D} | \psi_{IR} \rangle$$

$$= e^{-2\Delta u^*} \langle \psi_{IR} | e^{i(u_{IR}-u^*)D} \mathcal{O}(\epsilon) \mathcal{O}(0) e^{-i(u_{IR}-u^*)D} | \psi_{IR} \rangle$$

$u^* = \ln (x/\epsilon)$ gives the usual two-point function $\sim x^{-2\Delta}$
Twist fields (d=1)

$\Phi_n(x)$ almost local field, implements a branch point

$|\psi\rangle_n = \bigotimes^n |\psi\rangle$, $\rho_{[0,x]} = \text{tr}(-\infty,0) \cup (x,\infty) (|\psi\rangle\langle\psi|)$

$$\langle \Phi_n(x)\Phi_n(0) \rangle_n = \text{tr}(\rho_{[0,x]}^n)$$

Two key facts: (BGS '12)

1. $|\psi\rangle$ unentangled $\rightarrow \langle \Phi_n(x)\Phi_n(0) \rangle_n = 1$
2. $\Phi_n$ scaling field, $\Delta_n = \frac{c}{12} \left( n - \frac{1}{n} \right)$
Entanglement per scale

\[ S(u) \equiv S([0, x], e^{-iuD} |\psi_{IR}\rangle) \]

Using twist fields ...

\[ \text{tr}(\rho^n_{[0,x]})(u) = e^{-2u\Delta_n} \quad (u < \ln(x/\epsilon)) \]

Using the definition of S we find ...

\[ S(u = 0) = 0 \]
\[ \frac{dS(u)}{du} = \frac{c}{3}, \quad (u < \ln(x/\epsilon)) \]
\[ \frac{dS(u)}{du} = 0, \quad (u > \ln(x/\epsilon)) \]
What have we learned?

• We can give a formal derivation in the continuum language that each RG step adds definite entanglement (d=1)
• The process is more or less local in space and RG direction
• Entanglement and renormalization combine into a very beautiful variational state with similarities to holography

So perhaps entanglement is the fabric of spacetime/quantum matter
Some future attractions

• More about large N and strong coupling

• Fermi surfaces and non-Fermi liquids

• Time dependence

• More covariant formulations?
Large $N/strong$ coupling

- There should be something special about this limit ...
- How to see various kinds of bulk locality?
- Each node of the graph is like a chunk of AdS of size $\sim L_{AdS}$?
- Can we get the special spectrum of operator dimensions?

Yes! using “quantum expanders”:

$$\mathcal{E}(\rho) = \frac{1}{D} \sum_{\alpha=1}^{D} V_\alpha \rho V_\alpha^\dagger$$

Relation to Verlinde error correction scheme?
\[ ds^2 = dS^2 \]

Thank you!