dynamics of broken symmetry

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Simons Symposium - Quantum Entanglement
Caneel Bay, USVI
• 1957: J. R. Oppenheimer purchases small plot of land in Hawksnest bay - “Oppenheimer beach”

• 2013: Oppenheimer-Volkoff approximation returns to St John
[1207.4194 & PRL] in collaboration with

Jerome Gauntlett & Toby Wiseman

Benjamin Simons

Miraculous Joe Bhaseen
motivation: non-equilibrium dynamics

- The challenge: how to characterise **quantum dynamics** far from equilibrium?

- Lack of broadly applicable **principles** and **techniques**

- Progress has been achieved in **integrable** models:
  - quench to CFT: Cardy & Calabrese PRL 96 2006
  - quench in transverse Ising chain: Calabrese, Essler & Fagotti PRL 106 (2011)
  - quench in matrix models [arXiv today by Mandal & Morita]

- **Holography** provides non-integrable yet solvable examples
  - plasma quench: Chesler & Yaffe PRL 102 (2009)
  - vaidya quench: de Boer et al. PRL 106 (2011)
  - ... many more
We will endeavour to model QCPs using holography. Scaling symmetries encoded as isometries of `dual’ spacetime

Continuum theory near QCP is encoded in dynamics of dual string (really gravity) theory

Use the simplest holographic model which allows modelling dynamics of order parameter at finite density

Dual to a putative (matrix) large-N conformal gauge theory. Broken by finite density, temperature. No susy.

aim for generic results & lessons for out-of-equilibrium physics
1. motivation
   "holography provides solvable examples"

2. background
   "holographic dualities, equilibrium issues"

3. a holographic setup for dynamical symmetry breaking
   "Numerical relativity, structure of collective modes"

4. conclusions and outlook
   "dynamical consequences of symmetry breaking"
2. background

“holographic dualities, equilibrium issues”
gauge-gravity duality

- Duality between Anti-de Sitter gravity and conformal field theory (AdS\textsubscript{D+1}: very simple curved space, whose symmetries are given by the conformal group in D dimensions)

- In fact: CFT is just an RG fixed point — gauge-gravity duality describes whole RG flows between conformal (scale!)-invariant fixed points

Quantum theory of gravity on AdS\textsubscript{D+1} space
\[ \approx \]
Quantum field theory in D (flat) dimensions
learning from gauge-gravity duality

- Gravity in fully quantum regime is out of reach, but **semiclassical** is fine. It turns out, this is extremely **useful**

Semi-classical theory of gravity on AdS\(_{D+1}\) space

\[
\cong
\]

Strongly coupled large-N quantum field theory in D (flat) dimensions

- Time dependence in field theory translates to evolving Einstein equations + matter to construct bulk ⇒ well-defined problem even far from equilibrium
ensembles in gauge-gravity

- $Z_{\text{gravity}}$ is **generating functional** of CFT correlations. Incorporate finite temperature and density in formalism.

**Diagram:**
- UV (e.g. CFT$_3$)
- IR (e.g. BH)
- Holographic direction $\sim$ RG
- $\hat{O}(t, x)$
- $\psi_{\hat{O}}(z; t, x)$
- Boundary conditions on $Z_{\text{gravity}}$ determine ensemble
- Temperature: BH in bulk
- Charge density: flux from bulk BH
requirements on the model

• Want bulk solutions that allow for: charged BHs (notion of temperature), U(1) gauge field (chemical potential), complex scalar field (order parameter)


\[
\begin{align*}
gravity - metric \: g & \quad \leftrightarrow \quad energy-momentum \: T \\
gauge \: field \: A & \quad \leftrightarrow \quad conserved \: current \: J \\
charged \: scalar \: \psi & \quad \leftrightarrow \quad order \: parameter \: <O\psi>
\end{align*}
\]

• **Equilibrium** solutions are given by **static charged black holes**. At low temperatures symmetry is broken. Equilibrium phase diagram determined by finding all static solutions with right boundary conditions

• **Full evolution** of all the bulk fields allows to compute full dynamical study of order parameter formation/relaxation, thermalisation, ...
a holographic model of superconductivity

• We discuss a specific model of holographic superfluid. New results here in a dynamical context **more general** and may extend beyond holography

• Specific example: minimal model of holographic superconductor

\[
S = \int d^4x \sqrt{-g} \left[ R + \frac{6}{\ell^2} - \frac{1}{4} F^2 - |D\psi|^2 - m^2 |\psi|^2 \right]
\]

• Complex scalar \( \Psi \) is dual to symmetry-breaking order parameter
  1) RN: un-condensed normal phase, new hairy BH: s.c. phase
  2) leading near-boundary term of \( \Psi = \text{source} \); subleading term = vev
equilibrium phase diagram

- Construct homogeneous bulk solutions with \textit{spontaneous} $U(1)$ breaking → black hole with `scalar hair’

- finite chemical potential
- fixed $\psi^{(1)}=0$
- broken branch has \textit{lower} free energy
- 2\textsuperscript{nd} \textit{order} phase transition

\begin{itemize}
  \item $T=0$ limit of broken phase
  \item $T=0$ limit of unbroken phase
\end{itemize}

`standard’ black hole

\[ T/T_C \]
ads/cft dynamics: numerical relativity

- We wish to model a quench **holographically**: prescribe a sudden change in some physical parameter of the theory on the **boundary** and then evolve the non-linear PDEs numerically to ‘**fill in the bulk**’
outline

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3. a holographic setup for dynamical symmetry breaking
   “Numerical relativity, structure of collective modes”

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   “dynamical consequences of symmetry breaking”
3. a holographic setup for dynamical symmetry breaking

“Numerical relativity, structure of collective modes”
schematic of our setup

initial state $t = t_i$

boundary condition / quench $t = t_0$

final state at effective temperature $t = t_f$

non-linear non-equilibrium evolution

Quench!

static holographic superfluid

approach
more details of the setup [related work: Murata, Kinoshita & Tanahashi, 2010]

• for simplicity: take homogeneous quench of fixed width and variable height

  \[ J_0(t) = \delta e^{- (t/\bar{\tau})^2} \]

• the complex scalar can be expressed as

  \[ \psi(v, z) = z \left( \psi_1(v) + \hat{\psi}(v, z) \right) \]

• and \( \Psi_1(t) \) is the source \( J_0(t) \) at the boundary. Use source to quench the system (future work: different systems and different quenches...)

• solve system of (1+1) non-linear PDE by a pseudo-spectral method in spatial directions and ‘Crank-Nicholson’ finite differences in time direction (subtle issue about gauges. trial and error leads to stable choice)
the resulting dynamics I 

\[ \psi_1(t) = \delta e^{-(t/\bar{\tau})^2} \]

- The dynamics of this quench give rise to three distinct regimes

I. Oscillation

II. Decay to finite gap

III. Decay to zero gap

\[ \langle |\psi(t)|^2 \rangle / \langle |\psi_0|^2 \rangle \]

\[ \mu_f t \]

\[ \delta = 0.10 \]

\[ \delta = 0.19 \]

\[ \delta = 0.27 \]
the resulting dynamics II \( \psi_1(t) = \delta e^{-(t/\bar{\tau})^2} \)

- we can dress the results up as a dynamical phase diagram

- three regimes in nonequilibrium phase diagram. Using information of \( T_f(\delta) \) can collapse on equilibrium phase diagram (just a check). Underlying physics?
Gap dynamics in BCS theory [Levitov and others]

• BCS theory is the celebrated microscopic explanation of conventional superconductivity. An old story...

\[ \mathcal{H} = \sum_{p,\sigma} \epsilon_p a_{p,\sigma}^\dagger a_{p\sigma} - \frac{\lambda(t)}{2} \sum_{q,p} a_{p\uparrow}^\dagger a_{-p\downarrow}^\dagger a_{-q\downarrow} a_{q\uparrow} \]

\[ |\Psi(t)\rangle = \prod_p \left[ u_p(t) + v_p(t) a_{p\uparrow}^\dagger a_{-p\downarrow}^\dagger \right] |0\rangle \]

pairing gap: \[ \Delta(t) = \lambda \sum_p u_p(t) v_p^*(t) \]

• Recent (2004 - ) new developments: coupling quench ⇒ the resulting (collisionless, non-adiabatic) dynamics can be solved as a non-linear integrable system!
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**time-dependent BCS pairing problem**

\[ |\Psi(t)\rangle = \prod_p \left[ u_p(t) + v_p(t) a_{p\uparrow}^\dagger a_{-p\downarrow}^\dagger \right] |0\rangle \]

**pairing gap:**

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\[
\mathcal{H} = \sum_{p,\sigma} \epsilon_p a_{p,\sigma}^\dagger a_p\sigma - \frac{\lambda(t)}{2} \sum_{q,p} a_{p,\uparrow}^\dagger a_{-p,\downarrow}^\dagger a_{-q,\downarrow} a_{q,\uparrow}
\]

\[
|\Psi(t)\rangle = \prod_p \left[ u_p(t) + v_p(t) a_{p,\uparrow}^\dagger a_{-p,\downarrow}^\dagger \right] |0\rangle
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pairing gap: \[ \Delta(t) = \lambda \sum_p u_p(t)v_p^*(t) \]

- Recent (2004 - ) new developments: coupling quench ⇒ the resulting (collisionless, non-adiabatic) dynamics can be solved as a non-linear integrable system!
compare to (simplified) BCS [Levitov and others]

- The dynamics of this quench give rise to three distinct regimes

I. Oscillation

II. Decay to finite gap

III. Decay to zero gap

- our achievement is twofold: 1) we exhibit analogous phenomena in a strongly-coupled system including the issue of thermalization
  2) we identify a new and generic mechanism within dynamical symmetry breaking leading to this behaviour
considering asymptotic states

- at late-times the system approaches a **static** equilibrium solution. Solution must be located on family of time-independent black holes

- one can study the **approach** to such states in perturbation theory. Perturbations contain information about **collective modes** of dual system. We focus on the sector that couples to condensate fluctuation

- **normalisable** perturbations of the final state with causal boundary conditions - these are called **quasi-normal modes** in the language of gravity

\[ \delta \Phi_I(v, z) = e^{-i\omega v} \Phi_\omega(z) \]

- The **analytic structure** of the \( \Phi \) tells us about a) late-time behaviour of observables b) poles in n-point functions of dual operators
quasi-normal modes: detailed structure

order-parameter dynamics: \[ |\langle O(t)\rangle| = |\langle O_f \rangle + ce^{-i\omega_L t}| \]

\[ T > T_c \]

\[ T = T_c \]

\[ T_\ast < T < T_c \]

\[ T = T_\ast \]

\[ T < T_\ast \]
region III ($T>T_c$): pure decay to zero order parameter

- dominant QNMs have
  \[
  \text{Re}(\omega) \neq 0 \quad \text{Im}(\omega) \neq 0
  \]
- but $\langle \mathcal{O}_f \rangle = 0$ so

\[
|\langle \mathcal{O}(t) \rangle| = |\langle \mathcal{O}_f \rangle + ce^{-i\omega t}| = |c|e^{\text{Im}(\omega)t}
\]
region II ($T^* < T < T_c$): pure decay to nonzero order parameter

- dominant QNMs have
  \[ \text{Re}(\omega) = 0 \quad \text{Im}(\omega) \neq 0 \]

- now \( \langle \mathcal{O}_f \rangle \neq 0 \) so

\[
|\langle \mathcal{O}(t) \rangle|^2 = |\langle \mathcal{O}_f \rangle + ce^{-i\omega t}|^2 \\
= |\langle \mathcal{O}_f \rangle|^2 + |c|^2 e^{2\text{Im}(\omega)t} + 2|c| e^{\text{Im}(\omega)t} \text{Re} [\langle \mathcal{O}_f \rangle c^*]
\]
region I \((T<T^*)\): oscillatory decay to nonzero order parameter

- dominant QNMs have
  \[
  \text{Re}(\omega) \neq 0 \quad \text{Im}(\omega) \neq 0
  \]

- now \(\langle \mathcal{O}_f \rangle \neq 0\) so

\[
|\langle \mathcal{O}(t) \rangle|^2 = |\langle \mathcal{O}_f \rangle + ce^{-i\omega t}|^2 = |\langle \mathcal{O}_f \rangle|^2 + |c|^2 e^{2\text{Im}(\omega)t} + 2e^{\text{Im}(\omega)t}\left(\text{Re}[\langle \mathcal{O}_f \rangle c^*] \cos(\text{Re}(\omega)t) - \text{Im}[\langle \mathcal{O}_f \rangle c^*] \sin(\text{Re}(\omega)t)\right)
\]
dynamics of symmetry breaking

• T-reversal invariance means collective mode spectrum (manifested in our example as QNMs) must be symmetric under 

\[ \omega \rightarrow -\omega^* \]

• Poles in spectral function (and other observables) come in two varieties:
  a) pairs of poles off imaginary axis
  b) single poles on imaginary axis

1. S.c. phase transition: coalescence of two poles at TC at \( \omega = 0 \)

2. Broken U(1) \( \Rightarrow \) Single pole (i.e. mode) at \( \omega = 0 \) (Goldstone mode)

3. At T=0 no source of dissipation \( \Rightarrow \) leading poles are oscillatory in nature

\[ 1 + 2 + 3 = \text{BL dynamical phase diagram!} \]
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conclusions

• very interesting far-from-equilibrium problems are accessible at the intersection of numerical relativity and AdS/CFT.

• simulated a quantum quench in ads/cft: persistence of BL phenomena to strong coupling and in systems that thermalise makes it more likely to be observed in actual experiments

• in fact: our analysis shows that BL-type behaviour could be generic for dynamical breaking of a continuous symmetry. This makes the experimental point even more emphatically.

• What is the meaning and / or origin of the (higher) quasinormal modes in the dual theory?
thanks for your attention!