Quantum Entanglement: from quantum matter to string theory

Caneel Bay, February 4-8, 2013

Subir Sachdev
Modern phases of quantum matter
Not adiabatically connected
to independent electron states:

*many-particle*

*quantum entanglement*
Gapped quantum matter

*Spin liquids, quantum Hall states....*

Conformal quantum matter

*Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene*

Compressible quantum matter

*Strange metals in high temperature superconductors, Bose metals*
Gapped quantum matter

Spin liquids, quantum Hall states....

Vishwanath, Wen, Senthil, Oshikawa

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Myers, Klebanov, Polchinski, Strominger, Swingle, Lee

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

Liu, Hartnoll, McGreevy, Silverstein, Huijse, Zaanen, Horowitz, Sonner, Trivedi, Kachru, Ooguri
Gapped quantum matter

Spin liquids, quantum Hall states....

Vishwanath, Wen, Senthil, Oshikawa

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals
$|\Psi\rangle \Rightarrow \text{Ground state of entire system,}$

$$\rho = |\Psi\rangle \langle \Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

**Entanglement entropy** $S_E = -\text{Tr} (\rho_A \ln \rho_A)$
Entanglement entropy of a band insulator

Band insulators

An even number of electrons per unit cell
Entanglement entropy of a band insulator

\[ S_E = aP - b \exp(-cP) \]

where \( P \) is the surface area (perimeter) of the boundary between A and B.
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

P. Fazekas and P. W. Anderson,
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle i j \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \text{Bohr magneton} = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \bullet \bullet = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

P. Fazekas and P. W. Anderson, 
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[
\begin{align*}
\begin{array}{c}
\bullet \quad \bullet = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\end{array}
\end{align*}
\]

Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \frac{1}{\sqrt{2}} \left( |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle \right) \]

Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \scriptsize \begin{array}{c}
\begin{array}{c}
\bullet \quad \bullet \\
\end{array}
\end{array}
\quad = \frac{1}{\sqrt{2}} \left( \left\langle \uparrow \downarrow \right| - \left| \downarrow \uparrow \right\rangle \right) \]

Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \mathcal{O} = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ = \frac{1}{\sqrt{2}} \left( \langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle \right) \]

“Electric” excitation

\[ S=1/2, \text{ charge 0} \]
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \begin{align*}
\begin{array}{cc}
\text{“Electric”} & \text{excitation} \\
S=1/2, & \text{charge 0}
\end{array}
\end{align*} \]
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

“Electric” excitation

\[ S=1/2, \quad \text{charge 0} \]

\[ \bullet \bullet = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \mathcal{O} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

“Magnetic” excitation \( S=0, \) charge 0
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[
\text{``Magnetic'' excitation} \\
\text{S=0, charge 0}
\]

\[
\begin{array}{c}
\text{\( \frac{1}{\sqrt{2}} \)} \\
\text{\( (\uparrow\downarrow) - (\downarrow\uparrow) \)}
\end{array}
\]
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \mathbf{\hat{S}}_i \cdot \mathbf{\hat{S}}_j \]

\[ \mathbf{\hat{S}}_{i,j} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

“Magnetic” excitation
S=0, charge 0
Mott insulator: Kagome antiferromagnet

Alternative view

Pick a reference configuration
Mott insulator: Kagome antiferromagnet

Alternative view

A nearby configuration
Mott insulator: Kagome antiferromagnet

Alternative view

Difference: a closed loop
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Entanglement in the $\mathbb{Z}_2$ spin liquid

Entanglement in the $\mathbb{Z}_2$ spin liquid

Sum over closed loops: only an even number of links cross the boundary between A and B

$S_E = aP - \ln(2)$

where $P$ is the surface area (perimeter) of the boundary between A and B.


Mott insulator: Kagome antiferromagnet


Strong numerical evidence for a $Z_2$ spin liquid
Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han¹, Joel S. Helton², Shaoyan Chu³, Daniel G. Nocera⁴, Jose A. Rodriguez-Rivera²,⁵, Collin Broholm²,⁶ & Young S. Lee¹

Gapped quantum matter

*Spin liquids, quantum Hall states*...

Vishwanath, Wen, Senthil, Oshikawa

Conformal quantum matter

*Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene*

Myers, Klebanov, Polchinski, Strominger, Swingle, Lee

Compressible quantum matter

*Strange metals in high temperature superconductors, Bose metals*

Liu, Hartnoll, McGreevy, Silverstein, Huijse, Zaanen, Horowitz, Sonner, Trivedi, Kachru, Ooguri
Gapped quantum matter

Spin liquids, quantum Hall states...

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Myers, Klebanov, Polchinski, Strominger, Swingle, Lee

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals
Superfluid-insulator transition


Ultracold $^{87}\text{Rb}$ atoms - bosons
Excitations of the insulator:

\[ S = \int d^2r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + (\lambda - \lambda_c) |\Psi|^2 + u \left( |\Psi|^2 \right)^2 \right] \]

Particles \sim \Psi^\dagger

Holes \sim \Psi

Density of particles = density of holes \Rightarrow

“Relativistic” field theory for \Psi:

\[ S = \int d^2 r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]
\[ S = \int d^2r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2 \]

Particles and holes correspond to the 2 normal modes in the oscillation of $\Psi$ about $\Psi = 0$.  

\[ \langle \Psi \rangle \neq 0 \]

Superfluid  

\[ \langle \Psi \rangle = 0 \]

Insulator
\[ S = \int d^2r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]

Nambu-Goldstone mode is the oscillation in the phase \( \Psi \) at a constant non-zero \( |\Psi| \).

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]

\( \lambda \)
\[ S = \int d^2r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2 \]

A conformal field theory in 2+1 spacetime dimensions: a CFT3

\[ \langle \Psi \rangle \neq 0 \]

Superfluid

\[ \langle \Psi \rangle = 0 \]

Insulator

\( \lambda_c \)
$$S = \int d^2r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left( |\Psi|^2 \right)^2$$

No well-defined normal modes, or particle-like excitations

Superfluid

Insulator
\[
S = \int d^2rd\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]
\]
\[
V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2
\]

Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

\[
\langle \Psi \rangle \neq 0 \quad \text{Superfluid}
\]
\[
\langle \Psi \rangle = 0 \quad \text{Insulator}
\]
\[
S = \int d^2 r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]
\]
\[
V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2
\]

Despite rapid decay, there is a well-defined Higgs “quasi-normal mode”. This is associated with a pole in the lower-half of the complex frequency plane.

\[\langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[\langle \Psi \rangle = 0 \quad \text{Insulator} \]

D. Podolsky, A. Auerbach, and D. P. Arovas, PRB 84, 174522 (2011).
\[ \mathcal{S} = \int d^2r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]


The Higgs quasi-normal mode is at the frequency

\[ \frac{\omega_{\text{pole}}}{\Delta} = -i \frac{4}{\pi} + \frac{1}{N} \left( \frac{16 (4 + \sqrt{2} \log (3 - 2\sqrt{2}))}{\pi^2} + 2.46531203396 i \right) + \mathcal{O}\left( \frac{1}{N^2} \right) \]

where \( \Delta \) is the particle gap at the complementary point in the “paramagnetic” state with the same value of \( |\lambda - \lambda_c| \), and \( N = 2 \) is the number of vector components of \( \Psi \). The universal answer is a consequence of the strong interactions in the CFT3
\[ S = \int d^2r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \]

V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2

\[
\frac{\omega_{\text{pole}}}{\Delta} = -i \frac{4}{\pi} + \frac{1}{N} \left( \frac{16 (4 + \sqrt{2} \log (3 - 2\sqrt{2}))}{\pi^2} + 2.46531203396 i \right) + \mathcal{O} \left( \frac{1}{N^2} \right)
\]

where \( \Delta \) is the particle gap at the complementary point in the “paramagnetic” state with the same value of \( |\lambda - \lambda_c| \), and \( N = 2 \) is the number of vector components of \( \Psi \).

The universal answer is a consequence of the strong interactions in the CFT3.
Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

Honeycomb lattice antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

**Fig. 1.**
(a) A motif of the honeycomb lattice antiferromagnet consists of two nearest-neighbor and six next-nearest-neighbor bonds.
(b) Illustration of the various cluster sizes all pointing to the phase boundary of the PVB phase for \( J_2 = 3.6 \). With \( J_2 = 0 \), we have the usual two-sublattice AF phase. A staggered field at the left end of the cylinder. As \( J_2 \) increasing, we find that the magnetization reduces to near zero for \( J_2 = 0.26 \). This phase transition point is larger than the classical limit value of \( \frac{1}{\sqrt{2}} \).

Honeycomb lattice antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]


Critical theory for photons and deconfined spinons:

\[ S_z = \int d^2r d\tau \left[ |(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right] \]

Néel order \( \sim z_\alpha^* \sigma_\alpha \beta z_\beta \); PVB order \( \sim \) monopoles in \( A_\mu \)

Honeycomb lattice antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]


 Critical theory for photons and deconfined spinons:

\[ S_z = \int d^2r d\tau \left[ \left| (\partial_\mu - iA_\mu) z_\alpha \right|^2 + s |z_\alpha|^2 + u (|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu \nu \lambda} \partial_\nu A_\lambda)^2 \right] \]

Critical point is a conformal gauge theory for SU(N>4) antiferromagnets. Evidence for long correlation lengths and emergent gauge fluctuations for SU(2)
\[ S = \int d^2 r d\tau \left[ |\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2 \]

A conformal field theory in 2+1 spacetime dimensions: a CFT3

\[ \langle \Psi \rangle \neq 0 \]
Superfluid

\[ \langle \Psi \rangle = 0 \]
Insulator
Classical vortices and Goldstone oscillations

Classical Boltzmann gas of particles and holes

Quantum critical

Superfluid

Insulator

$T_{KT}$

$T_c$

Monday, February 4, 13
CFT3 at $T>0$

Quantum critical

Superfluid

Insulator

$T_{KT}$

$\lambda_c$
Quantum critical dynamics

Quantum “nearly perfect fluid” with shortest possible local equilibration time, $\tau_{eq}$

$$\tau_{eq} = C \frac{\hbar}{k_B T}$$

where $C$ is a universal constant.

Response functions are characterized by poles in LHP with $\omega \sim k_B T/\hbar$.

These poles (quasi-normal modes) appear naturally in the holographic theory.

(Analogs of Higgs quasi-normal mode.)

Quantum critical dynamics

Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

\[ \sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)] \]

(Q is the “charge” of one boson)

Quantum critical dynamics

Transport co-efficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Momentum transport

\[
\eta = \frac{\text{viscosity}}{s} = \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1) ]
\]

Vector large-$N$ expansion for frequency-dependent conductivity of CFT3

$$\sigma(\omega) = \frac{Q^2}{\hbar} \sum \left( \frac{\hbar \omega}{k_B T} \right) ; \quad \Sigma \to \text{a universal function}$$

Vector large-$N$ expansion for frequency-dependent conductivity of CFT3

$$\sigma(\omega) = \frac{Q^2}{\hbar} \sum \left( \frac{\hbar \omega}{k_B T} \right); \quad \Sigma \rightarrow \text{a universal function}$$

Vector large-$N$ expansion for frequency-dependent conductivity of CFT3

$$\sigma(\omega) = \frac{Q^2}{h} \sum \left( \frac{\hbar \omega}{k_B T} \right); \quad \sum \rightarrow \text{a universal function}$$

Small $\omega$ and vector large $N$ limits do not commute at $T > 0$.

Field theories in \( d + 1 \) spacetime dimensions are characterized by couplings \( g \) which obey the renormalization group equation

\[
 u \frac{dg}{du} = \beta(g)
\]

where \( u \) is the energy scale. The RG equation is \textit{local} in energy scale, \textit{i.e.} the RHS does not depend upon \( u \).
Key idea: Implement $r$ as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

J. McGreevy, arXiv0909.0518
For a relativistic CFT in $d$ spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation $(i = 1 \ldots d)$

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$
Holography

This gives the unique metric

\[ ds^2 = \frac{1}{r^2} \left( -dt^2 + dr^2 + dx_i^2 \right) \]

This is the metric of anti-de Sitter space \( \text{AdS}_{d+2} \).
AdS/CFT correspondence
AdS/CFT correspondence

AdS\(_4\)  \hspace{1cm} R^{2,1} \hspace{1cm} \text{Minkowski}

Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe “outside”: i.e. the region is surrounded by an imaginary horizon.


Monday, February 4, 13
The entropy of this region is bounded by its surface area (Bekenstein-Hawking-’t Hooft-Susskind).

AdS/CFT correspondence

AdS$_4$\quad R$^{2,1}$\quad Minkowski

Minimal surface area measures entanglement entropy

Computation of minimal surface area yields

\[ S_E = aP - \gamma, \]

where \( \gamma \) is a shape-dependent universal number.

Entanglement entropy obeys \( S_E = aP - \gamma \), where \( \gamma \) is a shape-dependent universal number associated with the CFT3.

This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant.

\[ S_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right] \]
For every primary operator $O(x)$ in the CFT, there is a corresponding field $\phi(x, r)$ in the bulk (gravitational) theory. For a scalar operator $O(x)$ of dimension $\Delta$, the correlators of the boundary and bulk theories are related by

$$
\langle O(x_1) \ldots O(x_n) \rangle_{\text{CFT}} =
Z^n \lim_{r \to 0} r_1^{-\Delta} \ldots r_n^{-\Delta} \langle \phi(x_1, r_1) \ldots \phi(x_n, r_n) \rangle_{\text{bulk}}
$$

where the “wave function renormalization” factor $Z = (2\Delta - D)$. 
For a U(1) conserved current $J_{\mu}$ of the CFT, the corresponding bulk operator is a U(1) gauge field $A_{\mu}$. With a Maxwell action for the gauge field

$$S_{M} = \frac{1}{4g_{M}^{2}} \int d^{D+1}x \sqrt{g} F_{ab} F^{ab}$$

we have the bulk-boundary correspondence

$$\langle J_{\mu}(x_{1}) \ldots J_{\nu}(x_{n}) \rangle_{\text{CFT}} = (Zg_{M}^{-2})^{n} \lim_{r \to 0} r_{1}^{2-D} \ldots r_{n}^{2-D} \langle A_{\mu}(x_{1}, r_{1}) \ldots A_{\nu}(x_{n}, r_{n}) \rangle_{\text{bulk}}$$

with $Z = D - 2$. 

**AdS/CFT correspondence**
A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the change in metric of the bulk theory. We write $\delta g_{\mu\nu} = (L^2/r^2)\chi_{\mu\nu}$, and then the bulk-boundary correspondence is now given by

$$
\langle T_{\mu\nu}(x_1) \ldots T_{\rho\sigma}(x_n) \rangle_{\text{CFT}} =
\left( \frac{Z L^2}{\kappa^2} \right)^n
\lim_{r \to 0} r_1^{-D} \ldots r_n^{-D} \langle \chi_{\mu\nu}(x_1, r_1) \ldots \chi_{\rho\sigma}(x_n, r_n) \rangle_{\text{bulk}},
$$

with $Z = D$. 

**AdS/CFT correspondence**
AdS/CFT correspondence

So the minimal bulk theory for a CFT with a conserved U(1) current is the *Einstein-Maxwell* theory with a cosmological constant

\[
S = \frac{1}{4g_M^2} \int d^4 x \sqrt{g} F_{ab} F^{ab} + \int d^4 x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right].
\]

This action is characterized by two dimensionless parameters: \( g_M \) and \( L^2/\kappa^2 \), which are related to the \( T = 0 \) conductivity \( \sigma(\omega) = \sigma_\infty \) and the central charge of the CFT.
This minimal action also fixes multi-point correlators of the CFT: however these do not have the most general form allowed for a CFT. To fix these, we have to allow for higher-gradient terms in the bulk action. For the conductivity, it turns out that only a single 4 gradient term contributes

\[ S_{\text{bulk}} = \frac{1}{g^2_M} \int d^4 x \sqrt{g} \left[ \frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] \]

\[ + \int d^4 x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right], \]

where \( C_{abcd} \) is the Weyl tensor. The parameter \( \gamma \) can be related to 3-point correlators of \( J_\mu \) and \( T_{\mu\nu} \). Both boundary and bulk methods show that \( |\gamma| \leq 1/12 \), and the bound is saturated by free fields.

D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247
There is a family of solutions of Einstein gravity which describe non-zero temperatures.

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right] \]
AdS/CFT correspondence at non-zero temperatures

\textbf{AdS}_4-\text{Schwarzschild black-brane}

\[ ds^2 = \left( \frac{L}{r} \right)^2 \left[ \frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right] \]

with \( f(r) = 1 - \left( \frac{r}{R} \right)^3 \)

A 2+1 dimensional system at its quantum critical point:
\[ k_B T = \frac{3\hbar}{4\pi R}. \]
AdS$_4$ theory of quantum criticality

Consequence of self-duality of Maxwell theory in 3+1 dimensions

Conductivity is independent of $\omega/T$ for $\gamma = 0$.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
AdS$_4$ theory of quantum criticality

\[ \frac{\sigma(\omega)}{\sigma_{\infty}} \]

\[ \gamma = \frac{1}{12} \]

\[ \gamma = 0 \]

\[ \gamma = -\frac{1}{12} \]

The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations.

The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations.

The $\gamma = 0$ case is the exact result for the large $N$ limit of SU($N$) gauge theory with $N = 8$ supersymmetry (the ABJM model). The $\omega$-independence is a consequence of self-duality under particle-vortex duality ($S$-duality).

AdS$_4$ theory of quantum criticality

\[ \frac{\sigma(\omega)}{\sigma_\infty} = \begin{cases} \frac{1}{12} & \gamma = 0 \\ -\frac{1}{12} & \gamma = \frac{1}{12} \end{cases} \]

- Stability constraints on the effective theory (|\gamma| < 1/12) allow only a limited $\omega$-dependence in the conductivity

Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada
(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature $T$. We find clear evidence for deviations from $\omega_k$ scaling of the conductivity towards $\omega_k/T$ scaling at low Matsubara frequencies $\omega_k$. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with $\omega/T$ at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5) Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.
Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature $T$. We find clear evidence for deviations from $\omega_k$ scaling of the conductivity towards $\omega_k/T$ scaling at low Matsubara frequencies $\omega_k$. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with $\omega/T$ at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.

QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

W. Witzack-Krempa and S. Sachdev, to appear
AdS$_4$ theory of quantum criticality

The holographic solutions for the conductivity satisfy two sum rules, valid for all CFT3s.

\[
\int_0^\infty d\omega \text{Re} \left[ \sigma(\omega) - \sigma(\infty) \right] = 0
\]
\[
\int_0^\infty d\omega \text{Re} \left[ \frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)} \right] = 0
\]

The second rule follows from the existence of an EM-dual CFT3.

Boltzmann theory chooses a “particle” basis: this satisfies only one sum rule but not the other.

Holographic theory satisfies both sum rules.

AdS$_4$ theory of quantum criticality

Poles in LHP of conductivity at $\omega \sim k_B T / \hbar$; analog of Higgs quasinormal mode–quasinormal modes of black brane

AdS$_4$ theory of quantum criticality

Poles in LHP of resistivity — quasinormal modes of S-dual theory

(a)$\mathcal{R}\{\sigma(w; \gamma = 1/12)\}$
(b)$\mathcal{R}\{\hat{\sigma}(w; \gamma = 1/12)\}$
(c)$\mathcal{R}\{\sigma(w; \gamma = -1/12)\}$
(d)$\mathcal{R}\{\hat{\sigma}(w; \gamma = -1/12)\}$

Gapped quantum matter
Spin liquids, quantum Hall states....
Vishwanath, Wen, Senthil, Oshikawa

Conformal quantum matter
Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene
Myers, Klebanov, Polchinski, Strominger, Swingle, Lee

Compressible quantum matter
Strange metals in high temperature superconductors, Bose metals
Liu, Hartnoll, McGreevy, Silverstein, Huijse, Zaanen, Horowitz, Sonner, Trivedi, Kachru, Ooguri
Gapped quantum matter
Spin liquids, quantum Hall states....

Conformal quantum matter
Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter
Strange metals in high temperature superconductors, Bose metals

Liu, Hartnoll, McGreevy, Silverstein, Huijse, Zaanen, Horowitz, Sonner, Trivedi, Kachru, Ooguri
Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$. Compressible systems must be gapless. Conformal systems are compressible at $\mu = 0$ in $d = 1$, but not for $d > 1$. We will obtain compressible states in holography by studying conformal field theories at a non-zero $\mu$. These are obtained by imposing the boundary condition $A_t(x, t, r!0) = \mu$. Monday, February 4, 13
Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

- Describe zero temperature phases where $d\langle Q\rangle/d\mu \neq 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H$, to $H - \mu Q$. 

Monday, February 4, 13
Compressible quantum matter

• Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

• Describe zero temperature phases where $\frac{d\langle Q\rangle}{d\mu} \neq 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H$, to $H - \mu Q$.

• Compressible systems must be gapless.
Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $d\langle Q \rangle/d\mu \neq 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H$, to $H - \mu Q$.

- Compressible systems must be gapless.

- Conformal systems are compressible at $\mu = 0$ in $d = 1$, but not for $d > 1$. 
Compressible quantum matter

• Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

• Describe zero temperature phases where $d\langle Q\rangle/d\mu \neq 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H$, to $H - \mu Q$.

• Compressible systems must be gapless.

• Conformal systems are compressible at $\mu = 0$ in $d = 1$, but not for $d > 1$.

• We will obtain compressible states in holography by studying conformal field theories at a non-zero $\mu$. These are obtained by imposing the boundary condition $A_t(x, t, r \to 0) = \mu$. 
One compressible state is the **solid** (or “Wigner crystal” or “stripe”).
This state breaks translational symmetry. Has integer number of particles per unit cell.
Another familiar compressible state is the \textbf{superfluid}.
This state breaks the global U(1) symmetry associated with $Q$

Condensate of fermion pairs
The only compressible phase of traditional condensed matter physics which does not break the translational or $U(1)$ symmetries is the Landau Fermi liquid.
The Fermi liquid

\[ \mathcal{L} = f^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f + 4 \text{ Fermi terms} \]

- Fermi wavevector obeys the Luttinger relation \( k_F^d \sim Q \), the fermion density.

- Sharp particle and hole of excitations near the Fermi surface with energy \( \omega \sim |q|^z \), with dynamic exponent \( z = 1 \).

- The phase space density of fermions is effectively one-dimensional, so the entropy density \( S \sim T \). It is useful to write this is as \( S \sim T^{(d-\theta)/z} \), with violation of hyperscaling exponent \( \theta = d - 1 \).
Logarithmic violation of “area law”: \[ S_E = \frac{1}{12} (k_F P) \ln(k_F P) \]

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A with an arbitrary smooth shape. The prefactor 1/12 is universal: it is independent of the shape of the entangling region, and of the strength of the interactions.

Logarithmic violation of “area law”: $S_E = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum $k_F$, where $P$ is the perimeter of region A with an arbitrary smooth shape. The prefactor $1/12$ is universal: it is independent of the shape of the entangling region, and of the strength of the interactions.

Logarithmic violation of “area law”: \( S_E = \frac{1}{12} (k_F P) \ln(k_F P) \)

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A with an arbitrary smooth shape. The prefactor \( 1/12 \) is universal: it is independent of the shape of the entangling region, and of the strength of the interactions.


Bosons with correlated hopping

\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijkl \in \square} b_i^\dagger b_k^\dagger b_j b_\ell \]
Bosons with correlated hopping

\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijkl \in \square} b_i^\dagger b_k^\dagger b_j b_\ell \]
Bosons with correlated hopping

\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijkl} b_i^\dagger b_k^\dagger b_j b_\ell \]
Bosons with correlated hopping

\[ H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijkl \in \square} b_i^\dagger b_k^\dagger b_j b_\ell \]

- **NFL Bose metal:** We have the fractionalization \( b \to f_1 f_2 \), where the \( f_1, f_2 \) both form a Fermi surface. Both fermions are gauge-charged, and so the Fermi surfaces are partially “hidden”.

\[ Q = b_1^\dagger b_1 \]

\[ A_f = \langle Q \rangle \]


S. Sachdev, arXiv:1209.1637
For suitable interactions, we can have the boson, \( b \), *fractionalize* into two fermions \( f_{1,2} \):

\[
b \rightarrow f_1 f_2
\]

This implies the effective theory for \( f_{1,2} \) is invariant under the U(1) gauge transformation

\[
f_1 \rightarrow f_1 e^{i\theta(x,\tau)}, \quad f_2 \rightarrow f_2 e^{-i\theta(x,\tau)}
\]

Consequently, the effective theory of the Bose metal has an emergent gauge field \( A_\mu \) and has the structure

\[
\mathcal{L} = f_1^\dagger \left( \partial_\tau - iA_\tau - \frac{(\nabla - iA)^2}{2m} - \mu \right) f_1 + f_2^\dagger \left( \partial_\tau + iA_\tau - \frac{(\nabla + iA)^2}{2m} - \mu \right) f_2
\]

The gauge-dependent \( f_{1,2} \) Green’s functions have Fermi surfaces obeying \( A_f = \langle Q \rangle \). However, these Fermi surfaces are not directly observable because it is gauge-dependent. Nevertheless, gauge-independent operators, such as \( b \) or \( b^\dagger b \), will exhibit *Friedel oscillations* associated with fermions scattering across these hidden Fermi surfaces.
FL
Fermi liquid

- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$. 
• $k_F^d \sim Q$, the fermion density

• Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$. 

• Hidden Fermi surface with $k_F^d \sim Q$. 

Monday, February 4, 13
FL
Fermi liquid

- \( k^d_F \sim Q \), the fermion density

- Sharp fermionic excitations near Fermi surface with \( \omega \sim |q|^z \), and \( z = 1 \).

- Entropy density \( S \sim T^{(d-\theta)/z} \) with violation of hyperscaling exponent \( \theta = d - 1 \).

- Entanglement entropy \( S_E \sim k^{d-1}_F P \ln P \).

NFL
Bose metal

- **Hidden** Fermi surface with \( k^d_F \sim Q \).

- Diffuse fermionic excitations with \( z = 3/2 \) to three loops.

A fluctuation at wavevector $q$ couples most efficiently to fermions near $\pm k_0$. 

\begin{itemize}
  \item A fluctuation at wavevector $q$ couples most efficiently to fermions near $\pm k_0$.
\end{itemize}
\[
\mathcal{L}[\psi_\pm, a] = \\
\psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \\
- a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left( \partial_y a \right)^2
\]
Simple scaling argument for $z = 3/2$. 

The Lagrangian is given by:

$$ \mathcal{L} = \psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- $$

$$ - a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2 $$

Field theory of non-Fermi liquid

Simple scaling argument for $z = 3/2$.

\[ \mathcal{L} = \psi_+^\dagger (\mathbf{\hat{A}}_r - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\mathbf{\hat{A}}_r + i\partial_x - \partial_y^2) \psi_- \\
- a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2 \]
Field theory of non-Fermi liquid

\[ \mathcal{L} = \psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \]

\[ - a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2 \]

Simple scaling argument for \( z = 3/2 \).

Under the rescaling \( x \to x/s, \ y \to y/s^{1/2}, \) and \( \tau \to \tau/s^z \), we find invariance provided

\[
\begin{align*}
    a & \to a \ s \\
    \psi & \to \psi \ s^{(2z+1)/4} \\
    g & \to g \ s^{(3-2z)/4}
\end{align*}
\]

So the action is invariant provided \( z = 3/2 \).

**FL**

**Fermi liquid**

- $k_F^d \sim Q$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

**NFL**

**Bose metal**

- Hidden Fermi surface with $k_F^d \sim Q$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

---

• $k_F^d \sim Q$, the fermion density

• Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

• Hidden Fermi surface with $k_F^d \sim Q$.

• Diffuse fermionic excitations with $z = 3/2$ to three loops.

• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$. 
**FL Fermi liquid**

- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

**NFL Bose metal**

- Hidden Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

- $S_E \sim k_F^{d-1} P \ln P$. 
Logarithmic violation of “area law”: \( S_E = C_E k_F P \ln(k_F P) \)

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A with an arbitrary smooth shape.

The prefactor \( C_E \) is expected to be universal but \( \neq 1/12 \):

- independent of the shape of the entangling region, and dependent
- only on IR features of the theory.

Holography

Huijse
Kachru
Liu
Swingle
Trivedi

$\mathcal{X}_i$

$r$
Consider a metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} \, ds. \]

Recall: conformal matter has \( \theta = 0, \, z = 1 \), and the metric is anti-de Sitter
Consider a metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

The value \( \theta = d - 1 \) reproduces all the essential characteristics of the entropy and entanglement entropy of a non-FL.

Consider a metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

The value \( \theta = d - 1 \) reproduces all the essential characteristics of the entropy and entanglement entropy of a non-FL.

Consider a metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

The null-energy condition of gravity yields \( z \geq 1 + \theta/d \). In \( d = 2 \), this leads to \( z \geq 3/2 \). Field theory on non-FL yields \( z = 3/2 \) to 3 loops!

This is a “bosonization” of the hidden Fermi surface
Holography of a non-Fermi liquid

Can we see the Fermi surface directly via “Friedel oscillations” in density (or related) correlations?

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015
Holography of a non-Fermi liquid

Can we see the Fermi surface directly via “Friedel oscillations” in density (or related) correlations?

Spatial dimension $d=1$

Monopoles in the 2+1 dimensional bulk U(1) gauge field acquire a Berry phase determined by the boundary U(1) charge density $Q$, and a dilute gas theory of monopoles leads to Friedel oscillations with

$$\langle \rho(x) \rho(0) \rangle \sim \frac{\cos(2k_F x)}{|x|^{2\Delta_F}}$$

T. Faulkner and N. Iqbal, arXiv:1207.4208

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015
Holography of a non-Fermi liquid

Can we see the Fermi surface directly via “Friedel oscillations” in density (or related) correlations?

Spatial dimension $d=1$

Monopoles in the 2+1 dimensional bulk U(1) gauge field acquire a Berry phase determined by the boundary U(1) charge density $Q$, and a dilute gas theory of monopoles leads to Friedel oscillations with

$$\langle \rho(x)\rho(0) \rangle \sim \frac{\cos(2k_F x)}{|x|^{2\Delta_F}}$$

T. Faulkner and N. Iqbal, arXiv:1207.4208

Exact solution of adjoint Dirac fermions at non-zero density coupled to a SU($N_c$) gauge field: low energy theory has an emergent $\mathcal{N} = (2, 2)$ supersymmetry, the global U(1) symmetry becomes the $R$-symmetry, and there are Friedel oscillations with

$$\Delta_F = 1/3 \quad \text{for all } N_c \geq 2$$

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via “Friedel oscillations” in density (or related) correlations?

Spatial dimension $d=2$

- For every CFT in 2+1 dimensions with a globally conserved U(1), we can define a monopole operator which transforms as a scalar under conformal transformations. 
  
  e.g. for the XY model, we insert a monopole at $x_m$ by including a fixed background gauge flux $\alpha_\mu$ so that

  $$\mathcal{L} = |(\partial_\mu - i\alpha_\mu)\psi|^2 + s|\psi|^2 + u|\psi|^4$$

  where the flux $\beta_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu \alpha_\lambda$ obeys

  $$\partial_\mu \beta_\mu = 2\pi \delta(x - x_m) \quad , \quad \epsilon_{\mu\nu\lambda} \partial_\nu (\Omega \beta_\nu) = 0$$

  where the CFT lives on the conformally flat space with is $ds^2 = \Omega^{-2} dx^2_{\mu}$.

S. Sachdev, arXiv:1209.1637
Holography of a non-Fermi liquid

Can we see the Fermi surface directly via “Friedel oscillations” in density (or related) correlations?

**Spatial dimension** $d=2$

- In the holographic theory, we have a bulk scalar field $\Phi_m$ (conjugate to the monopole operator of the CFT) which carries the charge of the $S$-dual of the 4-dimensional bulk U(1) gauge field:

$$S_m = \int d^4x \sqrt{-g} \left[ |(\nabla - 2\pi i \tilde{A})\Phi_m|^2 + \ldots \right]$$

where $\tilde{F} = d\tilde{A} = *F = *dA$.  

S. Sachdev, arXiv:1209.1637
Holography of a non-Fermi liquid

Can we see the Fermi surface directly via “Friedel oscillations” in density (or related) correlations?

Spatial dimension $d=2$

- When a chemical potential is applied to the boundary CFT, $\Phi_m$ experiences a magnetic flux. Consequently condensation of $\Phi_m$ leads to a vortex-lattice-like state, which corresponds to the formation of a crystal in the CFT. The crystal has unit $Q$ charge per unit cell.

- We expect that a vortex-liquid-like state of the $\Phi_m$ will yield the Friedel oscillations of the Fermi surface, with the correct Fermi wavevector.

S. Sachdev, arXiv:1209.1637

Kachru
Gapped quantum matter
Spin liquids, quantum Hall states....

Conformal quantum matter
Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter
Strange metals in high temperature superconductors, Bose metals