Book Review: *Quantum Phase Transitions*

Quantum Phase Transitions. S. Sachdev, Cambridge University Press, 1999.

Quantum phase transitions are defined as nonanalyticities in the ground state energy of a quantum system as a function of a certain coupling parameter in the system's Hamiltonian. These transitions are usually connected with profound changes in the nature of the correlations in the ground state, which is more specifically the subject matter of the book. It is a thorough and comprehensive attempt at an exposition of the main ideas and techniques in the field that has seen some thirty years of steady progress and has developed concepts and tools readily used also in other areas of physics. The book professes to be a graduate level text and can be used, at least certain chapters from it, as a very thorough course on quantum statistical mechanics, which should significantly widen its audience. It has without doubt been concieved with lots of pedagogical considerations thus making the subject approachable also by non-specialists such as myself. I found the author's remarks on the chapters suitable for newcomers quite helpful and found the parts of the book that I could follow in detail a delight to read.

The author specifically addresses second order quantum phase transitions which are characterized by the fact that the characteristic energy scale of fluctuations in the ground state vanishes at a critical value of the coupling parameter, or equivalently that the characteristic length scale diverges. Since experiments are usually performed at small, but nevertheless non-zero temperatures, it is very important to connect the consequences of the quantum singularity necessarily studied at $T=0$ with the properties of the system at non-zero temperatures. One hopes that the quantum singularity will leave a fingerprint also at finite temperatures and in deed the book explores this line of study quite thoroughly.

The author chooses two well understood systems as parade examples of quantum phases transitions: the quantum Ising model and the quantum

rotor model which are thoroughly analyzed and also have important experimental applications. Different aspects of the physics of these two systems make up for most of the part II of the book. These two examples also introduce the fundamental connection between quantum transitions in d dimensions and certain well studied finite temperature phase transitions in classical statistical mechanics in d + 1 dimensions. This is a realisation of utmost importance as it permitts to import the whole technology developed in classical statistical mechanics, most notably universality and the renormalization group method, to its quantum counterpart. The quantum-classical mapping leads to realization that classical correlation functions in d + 1 dimensions map onto quantum ones in d dimensions and one imaginary time direction.

Because of the universality brought forth by the quantum-classical mapping the properties of these quantum systems are described by an appropriate continuum quantum field theory since every second order quantum phase transition defines a quantum field theory in continuum. The line of developement of the book consists in understanding and classifying of different quantum field theories that arise from different quantum lattice Hamiltonians and later in describing the finite temperature properties of these quantum field theories which due to universality describe also the critical properties of the quantum Hamiltonians. Most of the part II of the book is dedicated to this program.

In view of this one legitimately raises the question of what is so special about quantum phase transitions after all. In principle they can be reduced to field theories and the results can be simply transfered from classical statmech to quantum phase transitions. In priciple yes, but the devil resides in the details of this program. The author discusses various important reasons why the quantum-classical mapping by itself is not enough to understand the quantum phase transitions and why a separate theory or theories are needed. The ponderousness of this book definitively testifies to that.

The book has three distinctive parts. Part I, being the introduction, basically surveys the field and gives it an experimental foundation. However, most of the chapters in Part II and Part III also contain a short Applications and Extensions sections that allow the reader to see the significance of the theoretical results in the real, i.e. experimental world.

Part II of the book deals with the quantum Ising and rotor models. Quantum Ising chain in a transverze field and the 1/N limit, the d = 2 and d = 3 limits of the rotor model are discussed thoroughly. This part of the book closes with a discussion of the properties close and above the upper critical dimension and a discussion of transport properties of the quantum rotor model in d = 2.

Part III surveys some important examples of quantum phase transitions in other models of physical interest: the boson Hubbard model, dilute Fermi and Bose gases, Fermi fluids, Heisenberg spins, bosonization in spin chains and disordered systems and spin glasses.

Some chapters are of course aimed at the specialist researchers in these respective fields. The book however contains chapters which are geared to a non-specialist graduate audience which are of course the ones that I enjoyed the most. The specialists might have their own opinion or criticism of this book but I find it extremely well written in the parts which do not wander too much off into the exotica. Some parts of the more general chapters I would definitively recommend to anybody giving a graduate course on quantum statistical mechanics. Overall a very thorough, interesting and well written book.

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