Anomalies and thermoelectric transport in a hydrodynamic theory of Weyl semi-metals

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Introduction

- Dirac and Weyl semi-metals have relativistic excitations near the Fermi energy.

- Relativistic theories can have chiral anomalies $\partial_\mu J^\mu \sim C\vec{E}\cdot\vec{B}$

- This leads to negative electrical magnetoresistance in Weyl semi-metals e.g.

$$\sigma \sim B^2 \tau$$

inter-node scattering time

Son & Spivak (2012)

- Chiral fermions have another anomaly: the mixed chiral-gravitational anomaly.

- Under appropriate conditions, the mixed anomaly produces anomalous thermoelectric and thermal conductivities.

Outline of Talk

- I want to explain what these anomalous effects are.
- I will do it by using a hydrodynamic model.
- Anomalies have subtle effects, which have been worked out in carefully for hydrodynamic theories.
- I will try to emphasise which results are consequences of hydrodynamics, and which are consequences of anomalies.
- Outline of the talk:

1). Anomalies in hydrodynamics

2). Transport in hydrodynamic model of Weyl semi-metals.

Relativistic hydrodynamics

- Hydrodynamics is the macroscopic theory of a system with local thermodynamic equilibrium.
- The fluid is described by $u^\mu(x^\mu),\ T(x^\mu),\ \mu(x^\mu)$

local velocity

local temperature

local chemical potential

- These are slowly varying functions: derivatives are small.
- This produces slowly varying hydrodynamic variables

$J^t(x^\mu)$ charge density

$T^{tt}(x^\mu)$ energy density

$T^{tx^i}(x^\mu)$ momentum density

- The hydrodynamic variables obey conservation equations

$$\partial_\mu J^\mu = 0 \qquad \partial_\mu T^{\mu\nu} = 0$$

in the absence of external sources ($F_{\mu\nu}$ and $g_{\mu\nu}$).

- Need to relate J^μ and $T^{\mu\nu}$ to u^μ , T and μ

Constitutive Relations I

- There are two steps to determine these constitutive relations.

1). Write the most general expression consistent with the symmetries

$$J^\mu = nu^\mu - T\sigma\left(\partial^\mu + u^\mu u_\nu \partial^\nu\right)\left(\frac{\mu}{T}\right) + \chi\left(\partial^\mu + u^\mu u_\nu \partial^\nu\right)T + O(\partial^2)$$

Write it as an expansion in powers of ∂_μ

2). Demand that there is an entropy current S^μ that obeys

$$\partial_\mu S^\mu \geq 0$$

when the hydrodynamic equations are satisfied.

- This restricts the values of some of the coefficients, e.g. $\chi = 0$

- A generating functional argument gives the same restrictions.

Banerjee et al ; Jensen et al (2012)

Constitutive Relations II

- This produces the constitutive relations (w/external sources)

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} - \eta\Delta^{\mu\rho}\Delta^{\nu\sigma}\left(\nabla_\rho u_\sigma + \nabla_\sigma u_\rho\right)$$

$$- (\zeta - \frac{2\eta}{3})\Delta^{\mu\nu}\nabla_\rho u^\rho + O(\nabla^2)$$

$$J^\mu = nu^\mu - \sigma_Q\Delta^{\mu\nu}\left(\nabla_\nu\mu - \frac{\mu}{T}\nabla_\nu T - F_{\nu\rho}u^\rho\right) + O(\nabla^2)$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

- This is the relativistic version of the Navier-Stokes equations.

- The coefficients depend on microscopic details of the theory.

- There is an entropy current $TS^\mu \equiv Pu^\mu - T^{\mu\nu}u_\nu - \mu J^\mu$

- Which obeys $\nabla_\mu S^\mu = \sigma T\nu_\mu\nu^\mu + \frac{1}{2}\eta\sigma_{\mu\nu}\sigma^{\mu\nu} + \frac{2\zeta}{3}\left(\nabla_\mu u^\mu\right)^2$

$$\nu^\mu = \Delta^{\mu\nu}\nabla_\nu\left(\frac{\mu}{T}\right) - \frac{F^{\mu\nu}u_\nu}{T} \qquad \sigma_{\mu\nu} = \Delta_{\mu\alpha}\Delta_{\nu\beta}\left(\nabla^\alpha u^\beta + \nabla^\beta u^\alpha\right) - \frac{2}{3}\Delta_{\mu\nu}\nabla_\delta u^\delta$$

The chiral anomaly

- Anomalies are when quantum effects violate the conservation laws.
- The chiral anomaly: $$\nabla_\mu J^\mu = -\frac{C}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$
- This violates the entropy production condition: $\nabla_\mu S^\mu \sim C\frac{\mu}{T}E\cdot B$
- To restore this, have to modify the constitutive relations e.g.

$$J^\mu \supset \frac{C\mu^2}{2}\left(1-\frac{2}{3}\frac{n\mu}{\epsilon+P}\right)\varepsilon^{\mu\nu\rho\sigma}u_\nu\nabla_\rho u_\sigma+\frac{C\mu}{2}\left(1-\frac{1}{2}\frac{n\mu}{\epsilon+P}\right)\varepsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}$$

Son & Surowka (2009)

- The anomaly has an effect even in the absence of an external electromagnetic field! The chiral vortical effect.
- This agrees with exact results found in holographic systems.

The mixed anomaly

- There is also a mixed chiral-gravitational anomaly G:

$$\nabla_\mu J^\mu = -\frac{C}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} - \frac{G}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}R^{\alpha}{}_{\beta\mu\nu}R^{\beta}{}_{\alpha\rho\sigma}$$

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu}J_\mu - \frac{G}{16\pi^2}\nabla_\mu\left(\epsilon^{\rho\sigma\alpha\beta}F_{\rho\sigma}R^{\nu\mu}{}_{\alpha\beta}\right)$$

- Set $R_{\mu\nu\rho\sigma} = 0$. But the anomaly still has an effect!

- It appears in the constitutive relations

$$J^\mu \supset -\frac{4G\mu n T^2}{\epsilon + P}\epsilon^{\mu\nu\rho\sigma}u_\nu\nabla_\rho u_\sigma - \frac{GT^2 n}{2\left(\epsilon + P\right)}\epsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}$$

- The coefficients are fixed by demanding that equilibrium properties can be derived from a generating functional (on a cone).

Jensen, Loganayagam & Yarom (2012)

- Again, this agrees with exact results in holographic systems.

Summary of anomaly effects

- In flat space, the anomalies change the conservation equations:

$$\nabla_\mu J^\mu = -\frac{C}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu}J_\mu$$

- And they produce an anomalous current

$$J^\mu \supset \left[\frac{C\mu^2}{2}\left(1-\frac{2n\mu}{3\left(\epsilon+P\right)}\right)-\frac{4G\mu nT^2}{\epsilon+P}\right]\epsilon^{\mu\nu\rho\sigma}u_\nu\nabla_\rho u_\sigma$$

$$+\left[\frac{C\mu}{2}\left(1-\frac{n\mu}{2(\epsilon+P)}\right)-\frac{GT^2n}{2\left(\epsilon+P\right)}\right]\epsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}$$

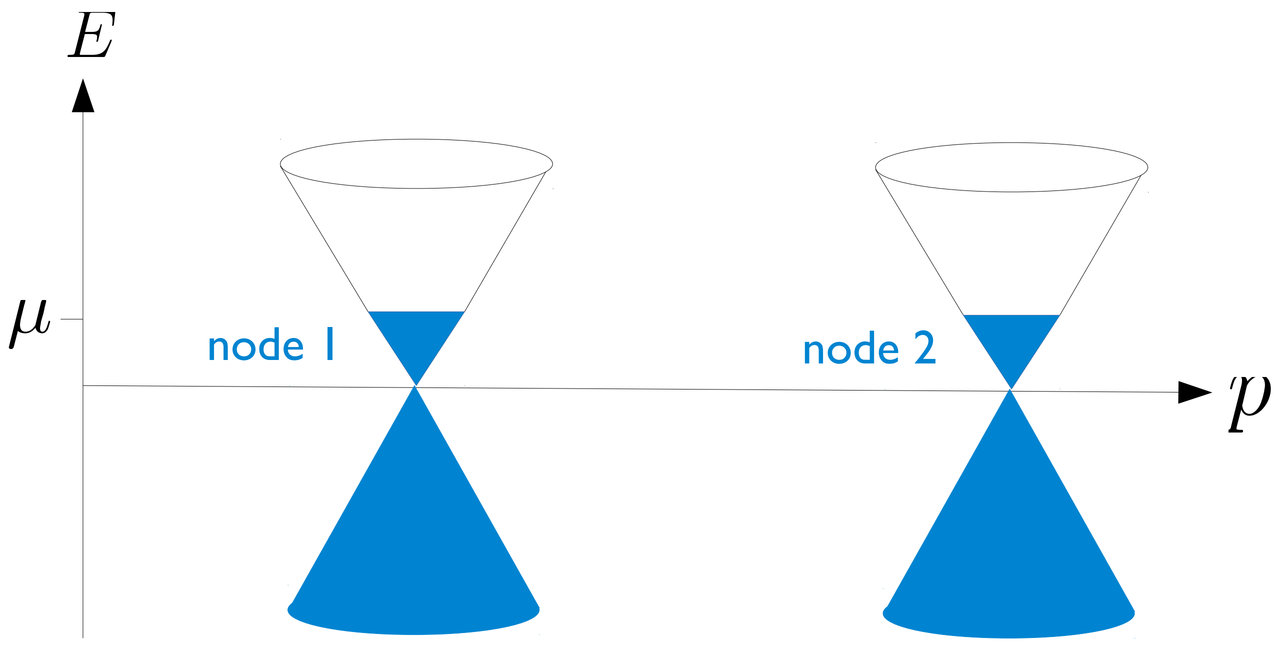
- For a Weyl node with Berry curvature k,

$$C = \frac{k}{4\pi^2} \qquad G = \frac{k}{24}$$

Landsteiner (2013)

Hydro model of a Weyl metal

- A metal with multiple Weyl nodes, with no net chirality:



$$\sum_a C_a = 0$$

$$\sum_a G_a = 0$$

- Treat each node as an independent chiral fluid.

- Introduce slow internode scattering:

$$\nabla_\mu J^\mu_a = -\frac{C}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} - \sum_b\left(\mathcal{R}_{ab}\frac{\mu_b}{T_b} + \mathcal{S}_{ab}\frac{1}{T_b}\right)$$

$$\nabla_\mu T^{\mu\nu}_a = F^{\nu\mu}J_\mu + u^\nu_a\sum_b\left(\mathcal{U}_{ab}\frac{\mu_b}{T_b} + \mathcal{V}_{ab}\frac{1}{T_b}\right)$$

Transport in hydro Weyl metal

- We want to compute the response to external electric fields and temperature gradients

$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & T\alpha_{ij} \\ T\bar{\alpha}_{ij} & T\bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\nabla_j T/T \end{pmatrix}$$

in the presence of a disordered chemical potential $\mu(\vec{x})$

and a magnetic field B

- We solved the hydro equations perturbatively at small disorder strength and small magnetic field.

- There are two contributions to each conductivity

$$J^i_a = n_a \delta v^i_a + B_i C_a \delta\mu_a \qquad Q^i_a = T s_a \delta v^i_a + 2TB_i G_a \delta T_a$$

- Bulk motion of the fluid & Anomalous effects parallel to B

Transport in hydro Weyl metal II

- 1). Bulk motion of the fluid produces a Drude-like response

$$\sigma_{zz} = \sum_a \frac{n_a^2}{\Gamma_a} \qquad \alpha_{zz} = \sum_a \frac{n_a s_a}{\Gamma_a} \qquad \bar{\kappa}_{zz} = \sum_a \frac{T s_a^2}{\Gamma_a}$$

due to relaxation of the fluid velocity (momentum).

Momentum relaxation rate $\Gamma_a = \dfrac{T^2 \left(s_a \left(\partial n_a / \partial \mu\right) - n_a \left(\partial s_a / \partial \mu\right)\right)^2}{3\sigma_{Qa} \left(\epsilon_a + P_a\right)^2}$

- 2). Anomalous conductivities, parallel to the B field

$$J_a^i = B^i C_a \delta\mu_a \qquad Q_a^i = 2T B^i G_a \delta T_a$$

- In the steady state, there are temperature and chemical potential differences between the nodes

e.g.
$$T^2 C_a B \delta E_z = \sum_b \left[\mathcal{R}_{ab} \left(T\delta\mu_b - \mu\delta T_b\right) - \mathcal{S}_{ab}\delta T_b\right]$$

- This is a balance of anomaly and inter-node scattering effects.

Transport in hydro Weyl metal III

- The result is anomalous contributions to the conductivities parallel to the magnetic field:

$$\sigma_{zz} = \mathfrak{s} B^2 \qquad \mathfrak{s} = T\left(C_a \;\; C_a\mu\right)\begin{pmatrix} \mathcal{R}_{ab} & -\mathcal{S}_{ab} \\ -\mathcal{U}_{ab} & \mathcal{V}_{ab} \end{pmatrix}^{-1}\begin{pmatrix} C_b \\ C_b\mu \end{pmatrix}$$

$$\alpha_{zz} = \mathfrak{a} B^2 \qquad \mathfrak{a} = 2T^2\left(0 \;\; G_a\right)\begin{pmatrix} \mathcal{R}_{ab} & -\mathcal{S}_{ab} \\ -\mathcal{U}_{ab} & \mathcal{V}_{ab} \end{pmatrix}^{-1}\begin{pmatrix} C_b \\ C_b\mu \end{pmatrix}$$

$$\bar{\kappa}_{zz} = \mathfrak{h} B^2 \qquad \mathfrak{h} = 4T^4\left(0 \;\; G_a\right)\begin{pmatrix} \mathcal{R}_{ab} & -\mathcal{S}_{ab} \\ -\mathcal{U}_{ab} & \mathcal{V}_{ab} \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ G_b \end{pmatrix}$$

- The chiral anomaly causes negative electric magnetoresistance.

- The mixed anomaly causes negative thermal magnetoresistance.

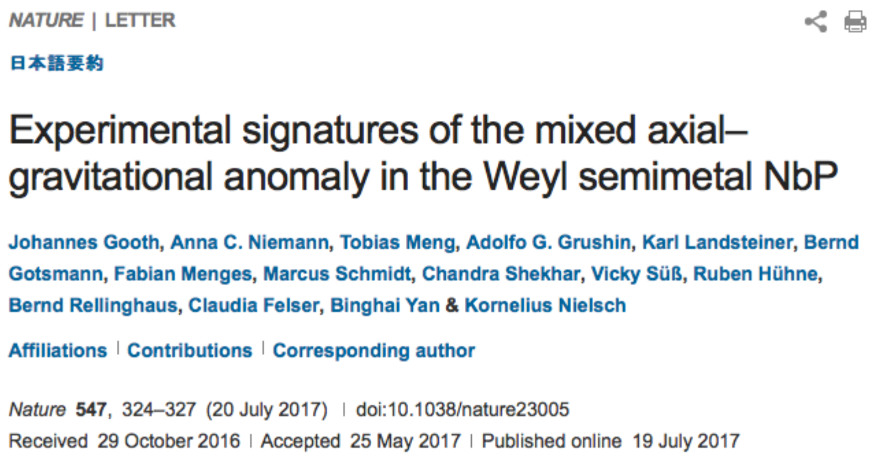
- Both together cause an anomalous thermoelectric conductivity.

- These are large for small internode scattering.

Experimental results

- Negative electrical magnetoresistance has been observed in various Weyl semi-metals.

- Anomalous thermoelectric conductivity recently seen in NbP:



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日本語要約

Experimental signatures of the mixed axial–gravitational anomaly in the Weyl semimetal NbP

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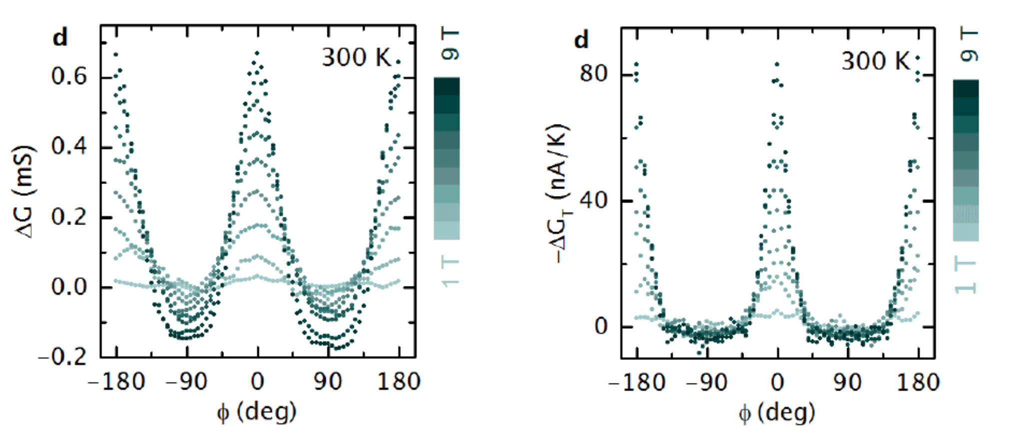

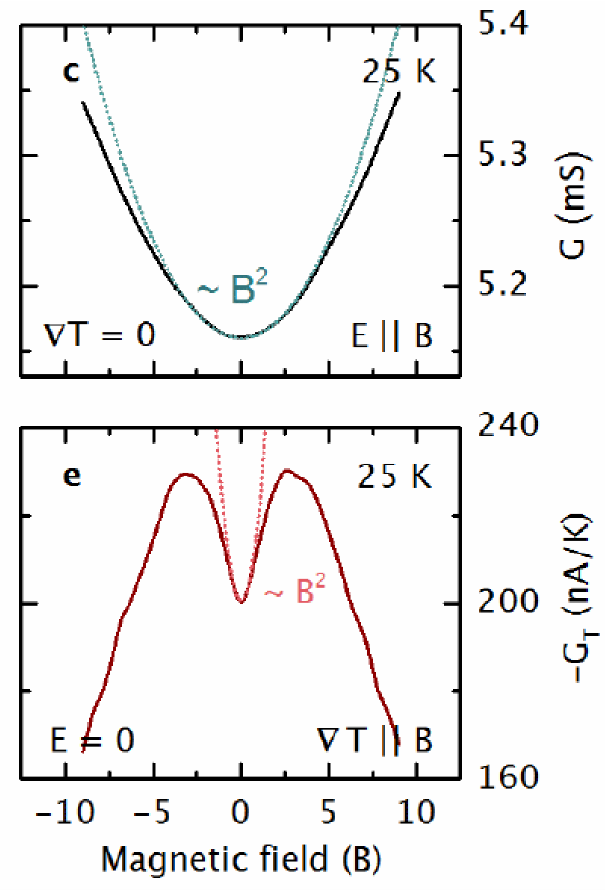

Conclusions

- There are two anomalies: chiral anomaly and the mixed chiral-gravitational anomaly.

- Even in flat space, the mixed anomaly is important.

- The mixed anomaly produces an anomalous thermal current in response to a temperature gradient.

- This results in anomalous thermoelectric and thermal conductivities under appropriate conditions.

- The model also has many other unusual properties due to its hydrodynamic nature,including violation of the WF law.

Non-hydrodynamic model

Gooth et al.

$$\partial_t \rho + \partial \cdot j = a_\chi E \cdot B \qquad \partial_t \varepsilon + \partial \cdot j_\varepsilon = j \cdot E$$

$$j = a_\chi \mu B \qquad j_\varepsilon = \left(\frac{a_\chi}{2} \mu^2 + a_g T^2 \right) B$$

$$\partial_t \varepsilon - \mu \partial_t \rho = -2 a_g T B \cdot \partial T - \tau^{-1} \left(\varepsilon - \mu \rho \right)$$

$$\partial_t \rho = 2 a_\chi B \cdot E - \tau^{-1} \rho$$