Scrambling Quantum Information in Cold Atoms with Light

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Quantum Information Scrambling

How fast can an initially localized quantum bit become entangled with all degrees of freedom, i.e., \textit{scrambled}?
Quantum Information Scrambling

How fast can an initially localized quantum bit become entangled with all degrees of freedom, i.e., scrambled?

**Inspiration:** information problem in black holes
Hayden, Preskill, Maldacena, Shenker, Susskind, Stanford …
Gauge/Gravity Duality

Quantum many-body system
\(d\) spatial dimensions

Spacetime geometry
\(d+1\) spatial dimensions

Figure adapted from Ramallo, arXiv:1310.4319v3[hep-th].
Gauge/Gravity Duality

Quantum many-body system

\[ d \] spatial dimensions

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Spacetime geometry

\[ d+1 \] spatial dimensions

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Can we realize quantum many-body systems in table-top experiments that are holographically dual to black holes? \textit{How would we know?}

Figure adapted from Ramallo, \textit{arXiv}:1310.4319v3[hep-th].
Fast Scrambling Conjecture

**Conjecture:** black holes are the fastest scramblers in nature

- Relaxation time $\tau = 1 / (2\pi T)$
- Scrambling time $t_s = \tau \log(S)$

$T = \text{Temperature}$

$S = \text{Entropy}$
**Conjecture:** black holes are the fastest scramblers in nature

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**Intuition:** random circuit model

- $S =$ number of qubits
- $\tau =$ interaction time
- Time $t_s \gtrsim \tau \log_2(S)$ to connect all pairs

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**Candidates for fast scrambling:** chaotic, *non-local* spin models

Outline

Background

Non-local interactions mediated by light
Quantifying many-body chaos

Prospects for Cold-Atom Experiments

Kicked top: intuitions from a simple model system
Non-local hopping & many-body chaos
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Photon-Mediated Interactions

optical cavity

cold atoms
Photon-Mediated Interactions

- **Non-local** → entangling atoms *en masse* for quantum metrology*
  
  → topological encoding of quantum information?*
  
  → novel quantum simulations: spin glasses*; *black holes?*

* Sorensen & Molmer (2002); MSS, Leroux & Vuletic (2010); Hosten … & Kasevich (2016).
* Gopalakrishnan, Lev; Sachdev; Diehl, …
Photon-Mediated Interactions

- **Non-local** → entangling atoms *en masse* for quantum metrology
  → topological encoding of quantum information?
  → novel quantum simulations: spin glasses; *black holes*?
- Easy to switch on/off and control sign
- Quantitative understanding of interaction-to-dissipation ratio
Photon-Mediated Spin Interactions

Two-level atom as pseudo-spin
Photon-Mediated Spin Interactions

Pairwise correlated spin flips:

\[ H \propto \sum_{i,j} (s_i^+ + s_i^-)(s_j^+ + s_j^-) \propto \sum_{i,j} s_x^i s_x^j \]

Sørensen & Mølmer, PRA (2002).
Photon-Mediated Spin Interactions

Pairwise correlated spin flips:

\[
H \propto \sum_{i,j}(s^i_+ + s^-_i)(s^j_+ + s^-_j) \propto \sum_{i,j} s^i_x s^j_x
\]

- Spatial addressing enables controlled interactions between arbitrary pairs
Photon-Mediated Spin Interactions

Pairwise correlated spin flips: \[ H \propto \sum_{i,j} (s^i_+ + s^i_-)(s^j_+ + s^j_-) \propto \sum_{i,j} s^i_+ s^j_+ \]

- Spatial addressing enables controlled interactions between arbitrary pairs
- Sign of interaction controlled by sign of detuning \( \delta \)
Photon-Mediated Spin Interactions

Pairwise correlated spin flips: \[ H \propto \sum_{i,j} (s^i_+ + s^-_i)(s^j_+ + s^-_j) \propto \sum_{i,j} s^i_x s^j_x \]

- Spatial addressing enables controlled interactions between arbitrary pairs
- Sign of interaction controlled by sign of detuning \( \delta \)
- Coherent interactions for \( \delta \gg \kappa \) and strong coupling \( \eta \equiv 4g^2/(\kappa \Gamma) \gg 1 \)
Experiment Design

• Strong coupling: \( \eta \equiv \frac{4g^2}{\kappa \Gamma} \sim \frac{F \chi^2}{w^2} \gg 1 \)

• Optical access for imaging & addressing

• Confinement in 3D lattice

\( \sim 10^1 - 10^3 \) atoms
Experiment Design

- Strong coupling: \( \eta \equiv \frac{4g^2}{\kappa \Gamma} \sim \frac{F \chi^2}{w^2} \gg 1 \)
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\(~ 10^1 - 10^3 \text{ atoms} \)

\( \Rightarrow \) Near-concentric resonator

Length \( L \sim 5 \text{ cm} \)
Waist \( w \sim 12 \mu \text{m} \)
Finesse \( F \sim 10^5 \)
Strong Coupling with Optical Access

Single-atom cooperativity $\eta \sim 50$

Cooperativity $\eta$

Finesse $6 \times 10^4$

F = $10^6$

F = $10^5$

F = $10^4$

Waist (\(\mu\)m)

Viewport

cavity
Atoms in the Cavity

Shift of the cavity resonance due to refractive index of a cloud of hundreds of atoms
Atoms in the Cavity

Image of atoms

Image of spin texture

$S_\Phi / S$

Shift of the cavity resonance due to refractive index of a cloud of hundreds of atoms
Photon-Mediated Spin Interactions

\[ H \propto \sum_{i,j} (s^i_+ + s^i_-)(s^j_+ + s^j_-) \propto \sum_{i,j} s^i_x s^j_x \]

Simple limit: all-to-all interaction

collective spin \( S = \sum_{i=1}^{N} s_i \)

\[ H = \chi S_x^2 \] twist
Spin Squeezing

Twisting strength $Q = N \chi t = \left(\text{# of photons scattered into cavity per atom}\right)$

$N = 4 \times 10^4$ atoms

$\eta = 0.1$, $\delta = \kappa / 2$
Global Spin Interactions

Cavity QED


Ion traps

Also: Monz, … & Blatt *PRL* (2011).

BECs


Vision: Non-Local Interactions

- NP-hard optimization problems

\[ H = \left( \sum_{i=1}^{N} w_i s_i^x \right)^2 \]

\[ \text{partition problem} \]

- Qubit-ensemble interface \( \Rightarrow \) Schrödinger cat states

- Non-local + chaotic \( \Rightarrow \) fast scrambling?

![Diagram with cavity, control light, and states](image-url)
Quantifying Scrambling

How to define chaos in a quantum many-body system?
Quantifying Scrambling

How to define chaos in a quantum many-body system?

**Quantum many-body butterfly effect**: growth of commutator $[V, W_t]$ between initially commuting operators vs. their separation in time $t$

How fast does $W_t = e^{-iHt} W e^{iHt}$ fail to commute with $V$ due to interactions $H$?

Measuring Fast Scrambling

Decay of out-of-time-order correlation function \( F(t) \equiv \langle \psi | W_t^\dagger V^\dagger W_t V | \psi \rangle \) indicates growth of commutator:

\[
\text{Re}[F(t)] = 1 - \langle |[W_t, V]|^2 \rangle / 2
\]
Measuring Fast Scrambling

Decay of out-of-time-order correlation function \( F(t) \) indicates growth of commutator:

\[
\text{Re}[F(t)] = 1 - \frac{\langle [W_t, V]^2 \rangle}{2}
\]

Where \( [V, W] = 0 \) at \( t=0 \) and \( V, W \) are simple operators, e.g., spin rotations.
Measuring Fast Scrambling

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Measuring Fast Scrambling

B. Swingle, G. Bentsen, MS-S, & P. Hayden,
PRA 040302(R) 2016.

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Tools for measuring \( F \)

- **Time reversal** \((H \rightarrow -H)\)
- **Many-body interferometry**

Also see:

Scrambling in a Cavity?

Photon-mediated interactions can enable...

- qubit-controlled operation
- switchable-sign interactions within ensemble

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Non-local spin models: candidates for fast scrambling

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Non-local spin models: candidates for fast scrambling

Globally interacting models: ease of visualization
• intuition: semiclassical limit
• numerical simulations

B. Swingle, G. Bentsen, MS-S, & P. Hayden, PRA 040302(R) 2016.

Chaotic “Kicked Top”
Chaotic Kicked Top

\[ H = \frac{k S^2}{2S} \sum_{n=\infty}^{\infty} \delta(t - n\tau) + \frac{\rho}{\tau} S_z \]

- Expect \( n_{chaos} \sim \log N \) kicks for initial state of solid angle \( \sim 1/N \) to spread over the entire \( N \)-atom Bloch sphere.

- Probe with rotations \( V = W = e^{i\phi S_z} \) by small angle \( \phi = 1/\sqrt{N} \).

Scrambling of a Kicked Top

Reference: time-ordered correlation function \( G = \langle V^\dagger_t V \rangle \)

Scrambling: out-of-time-order correlation \( F = \langle W^\dagger_t V^\dagger W_t V \rangle \)

\[ |G| \]

\[ |F|, |G| \]

Atom number \( N=2S \)

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- Scrambling time grows as $t_S \sim \log(N)$ $\leftrightarrow$ butterfly effect on Bloch sphere
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Reference: time-ordered correlation function \( G = \langle V_t^\dagger V \rangle \)

Scrambling: out-of-time-order correlation \( F = \langle W_t^\dagger V^\dagger W_t V \rangle \)

- Scrambling time grows as \( t_S \sim \log(N) \leftrightarrow \) butterfly effect on Bloch sphere
- Accessible for up to \( N \sim e^{\sqrt{\eta}} \sim 10^3 \) atoms at cavity cooperativity \( \eta = 50 \)
Scrambling Experiments

Kicked top with pseudo-spin $J = 5$:
“spin” states = momentum states of BEC


Twisting Hamiltonian of ~100 ions:
Multiple quantum coherence method


NMR experiments:

Wei, Ramanathan & Cappellaro,

cf. Davis, Bentsen, & MS-S PRL (2016).
Engineering Fast Scrambling?

All-to-all interactions restrict us to “single-particle” physics…

…but more complex non-local interactions should allow information to spread fast over exponentially large Hilbert space…

Can a single mode of light mediate more complex interactions?

Entropy:

\[ S \leq \ln(N) \]

\[ S \sim N \]
Exotic XY Models

Photon-mediated interactions for versatile control of long-range “hopping”:

\[ H_I = \sum_{i > j} J(i - j) \sigma_i^+ \sigma_j^- + \text{h.c.} \]

hard-core bosons = spin excitations: \( \bigcirc = \downarrow; \ \bullet = \uparrow \)
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Approach:

- Suppress hopping with magnetic field gradient
- Restore hopping at arbitrary distances \( i-j \) with modulated control field

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Approach:

- Suppress hopping with magnetic field gradient
- Restore hopping at arbitrary distances \( i-j \) with modulated control field
- Magnon dispersion relation = modulation waveform

Dispersion Engineering

Efficiently spread information over long distances by coupling $i^{th}$ spin to $i\pm 1, i\pm 2, i\pm 4, i\pm 8, \ldots, i \pm 2^l$
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$$E(k) = -2 \sum_{l=0}^{l_{\text{max}}} J_l \cos(2^l k - \phi_l)$$

$\text{control field spectrum}$

$E(k)$ graph with $l_{\text{max}} = 0$
Efficiently spread information over long distances by coupling $i^{th}$ spin to $i \pm 1, i \pm 2, i \pm 4, i \pm 8, \ldots, i \pm 2^l$

\begin{equation}
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\end{equation}

$\text{control field spectrum}$
Efficiently spread information over long distances by coupling \( i^{th} \) spin to \( i \pm 1, i \pm 2, i \pm 4, i \pm 8, \ldots, i \pm 2^l \).

\[
E(k) = -2 \sum_{l=0}^{l_{\text{max}}} J_l \cos(2^l k - \phi_l)
\]

\( l_{\text{max}} = 0 \)  \( l_{\text{max}} = 1 \)  \( l_{\text{max}} = 2 \)
Dispersion Engineering

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$$E(k) = -2 \sum_{l=0}^{l_{\text{max}}} J_l \cos(2^l k - \phi_l)$$

$\alpha = 0.686$ for $l_{\text{max}} = 1$

Dispersion relation is a fractal!
“Chaotic” dispersion?

\[ E(k) = -2 \sum_{l=0}^{l_{\text{max}}} J_l \cos(2^l k - \phi_l) \]

Looks crazy but must be \textbf{integrable}, since quasimomentum is conserved
“Chaotic” dispersion?

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\[ l_{\text{max}} = 0, 1, 2, 6 \]

\[ \text{Energy Level Statistics} \]

\[ \text{Poisson distribution of level spacings} \]
Engineered Chaos

Break integrability with disorder potential: \( H = \sum_i h_i \sigma_z^i + \sum_{i-j=2^l} J e^{i \phi_l} \sigma_+^i \sigma_-^j + \text{h.c.} \)

Signature of chaos: level repulsion

**Single particle**

- \( n=1 \)
- \( n=2 \)
- \( n=3 \)

**Strongly interacting**

- \( n=4 \)

**Diagrams:**
- N=9 sites, n=1 spins up
- N=9 sites, n=2 spins up
- N=9 sites, n=3 spins up
- N=9 sites, n=4 spins up

**Single hole**
Single-Particle vs. Many-Body Chaos?

Speed and depth of scrambling vs. boson number $n$?

Diagnostic: $F_V(t) = \langle V_t^\dagger V_0^\dagger V_t V_0 \rangle$ for two representative operators
Single-Particle vs. Many-Body Chaos?

$N = 8$ sites

Speed and depth of scrambling vs. boson number $n$?

Diagnostic: $F_V(t) = \langle V_t^\dagger V_0^\dagger V_t V_0 \rangle$ for two representative operators

$W = \text{nearest-neighbor swap}$

$U = \text{local phase shift}$

$1, 7$

$2, 6$

$3, 5$

$4$
Single-Particle vs. Many-Body Chaos?

Speed and depth of scrambling vs. boson number \( n \)?

Diagnostic: \( F_V(t) = \langle V_t^\dagger V_0^\dagger V_t V_0 \rangle \) for two representative operators

Deepest scrambling at half filling ("many"-body limit)… How deep?
Depth of Scrambling

How fully is the system scrambled at late times?

$$F \sim \frac{1}{(\text{Hilbert space dimension})}$$

$$\langle R[F]^2 \rangle$$

Dimension $$C_{N,n}$$

$$N = 10 \text{ sites}$$

$$F \sim \frac{1}{\text{dim}^2}$$
Depth of Scrambling

How fully is the system scrambled at late times?

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\[ \langle R[F]^2 \rangle \]

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**Depth of Scrambling**

$N = 10$ sites

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$\langle \mathcal{R}[F]^2 \rangle \\ \frac{1}{\text{dim}^2}$
Depth of Scrambling

How fully is the system scrambled at late times?

\[ F \sim \frac{1}{\text{Hilbert space dimension}} \]

\[ \langle R[F]^2 \rangle \propto \frac{1}{\text{dim}^2} \]

- **W** = nearest-neighbor swap
- **U** = local phase shift

⇒ *interactions (hard-core)* promote full scrambling
Fast Scrambling?

$N = 10$ sites

$F_U = \langle U_t \dagger U_0 \dagger U_t U_0 \rangle$

$U = \text{local phase shift}$

$F_{\text{min}} \sim 2^{-N}$ at half filling

Scrambling time: $t_S \sim \tau \log(N)$; $\tau = 1 / (2\pi T)$

Extending numerics will help a little…

Quantum simulations will help more!
Scrambling is the ultimate form of thermalization, predicted to be subject to a fundamental speed limit.

Which systems scramble fully and how fast? Many open questions… …ready to be tackled in cold-atom quantum simulations.

Photons can enable versatile engineering of interaction graphs:
• single-particle chaos amenable to semiclassical intuition
• interacting many-body chaos $\Rightarrow$ towards black-hole analogs?
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Extras
Numerical Simulation: Chaotic Kicked Top

Reference: time-ordered correlation function \( G = \langle V_t^\dagger V \rangle \)

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Atom number \( N=2S \)

More dissipation? Hard to calculate!