Diffusion and Chaos in Quantum Matter

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MAB -1603.08510 (PRL)
MAB -1604.01754
MAB and Donos - 1611.09380
MAB, Davison and Sachdev -1705.07896

Motivation

- Evidence has recently emerged for a new connection between transport (conductivities etc) and many-body chaos.
- Long-standing idea that dynamics at strong coupling governed by `Planckian' mean free time

$$\tau \sim \frac{\hbar}{k_B T}$$

Sachdev; Zaanen

- In a Fermi liquid thermoelectric diffusion constants are of form

$$D \sim v_F^2 \tau_{FL}$$

- Hartnoll (2014) proposed that strange metal transport is governed by

$$D \sim \frac{\hbar v^2}{k_B T}$$

- But unclear how to identify characteristic velocity of a strongly coupled theory.

Proposal

- A natural infra-red velocity is provided by speed at which chaos propagates

$$\langle [\hat{W}_x(t_w), \hat{V}_y(0)]^2 \rangle_\beta \sim f_1 e^{\lambda_L (t_w - t_* - |x-y|/v_B)}$$

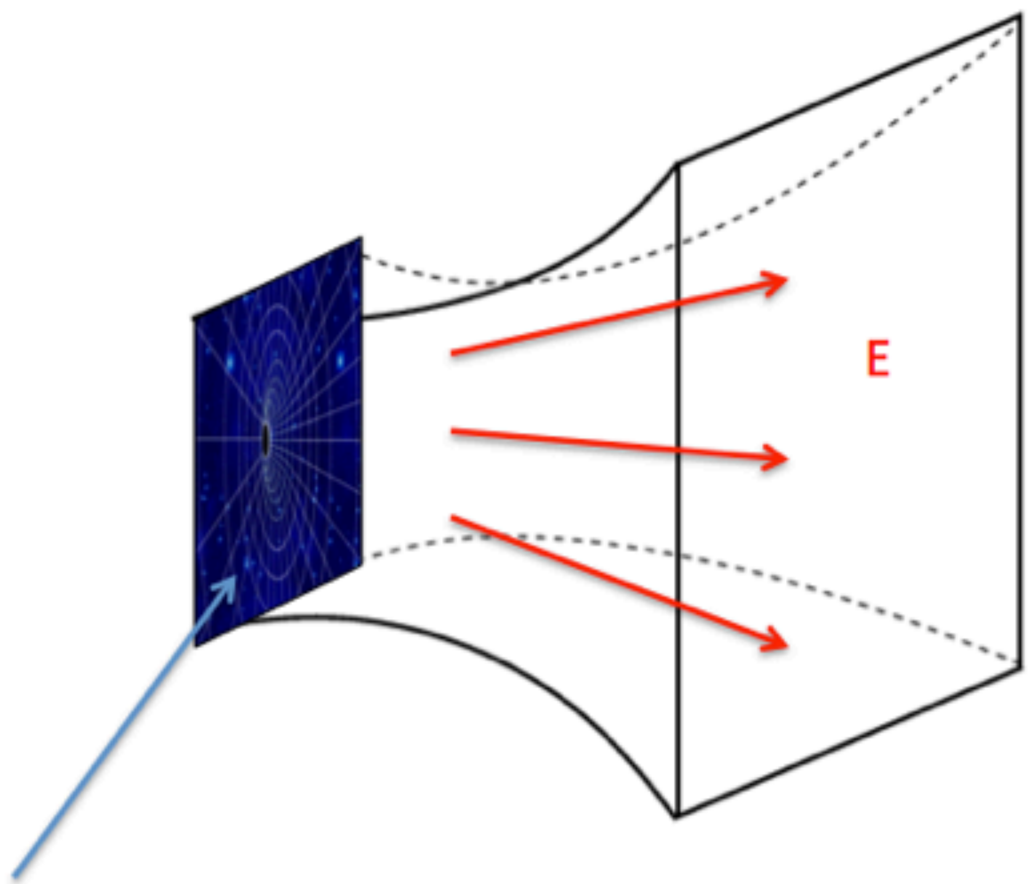
- Suggest butterfly velocity as characteristic velocity for diffusion

$$D \sim v_B^2 \tau$$

- In many cases also expect Lyapunov exponent to provide relevant mean free time $\tau \sim \tau_L = \lambda_L^{-1}$

Holography

Classical gravity in asymp-AdS spacetime ⟷ Strongly coupled large N gauge theory



Boundary field theory

- Reissner-Nordstrom black hole
- Hawking radiation = finite temperature, T
- Electric field = chemical potential, μ

- DC thermoelectric conductivities $\sigma \quad \alpha \quad \bar{\kappa}$ can be related to geometry and fields at black hole horizon.

MAB & Tong;
Donos & Gauntlett

- Likewise chaos exponents can be calculated from gravitational shock-wave on horizon.

Shenker & Stanford;
Roberts, Stanford & Susskind

- Diffusion constants proportional to conductivities through Einstein relations.

Holographic examples

- First evidence came from charge diffusion in particle-hole symmetric theories.

$$D_c = \frac{\sigma}{\chi} \qquad \chi = \left(\frac{\partial \rho}{\partial \mu}\right)_T$$

- Calculated this for holographic theories that flow to Lifshitz/hyperscaling geometries in IR.
- Generalised scaling theories described by critical exponents (z, θ, ϕ)

- Shock-wave calculation for these geometries gives

$$\tau_L^{-1} = 2\pi T \qquad v_B^2 \sim L^2 T^{2-2/z}$$

- Found these chaos parameters were universally related to the diffusion constant

$$\boxed{D_c = \frac{d_\theta}{\Delta_\chi} v_B^2 \tau_L}$$

MAB

- Although such a relationship is quite special to particle-hole symmetric theories.

- More general connection is found in relationship between energy/thermal diffusion and chaos

$$D_T = \frac{\kappa}{c_\rho} \qquad c_\rho = T\left(\frac{\partial s}{\partial T}\right)_\rho$$

- For these scaling geometries this is always given by

$$\boxed{D_T = \frac{z}{2z-2} v_B^2 \tau_L}$$

MAB;
MAB, Davison and Sachdev

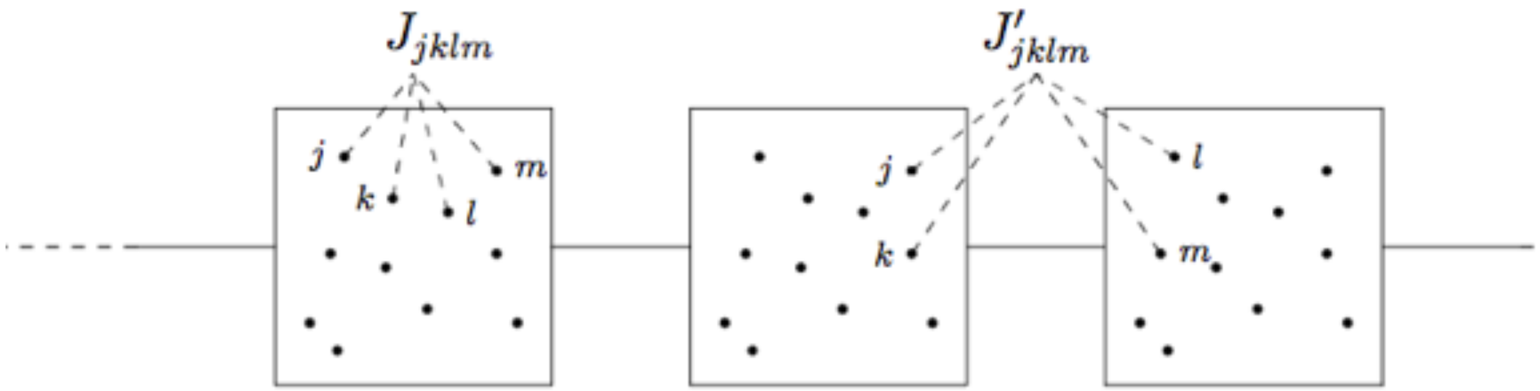
- This holds independently of charge density, magnetic field, periodic potential strength.

Fixed points with z=1 (e.g. CFTs) are a special exception

Other Examples

SYK chains

Gu, Stanford & Qi;
Davison et al



$$\tau_L^{-1} = 2\pi T$$

$$v_B^2 \sim \frac{J'^2 T}{J}$$

$$D_T = v_B^2 \tau_L$$

Critical Fermi surfaces

Quantum chaos on a critical Fermi surface

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Abstract

We compute parameters characterizing many-body quantum chaos for a critical Fermi surface without quasiparticle excitations. We examine a theory of N species of fermions at non-zero density coupled to a $U(1)$ gauge field in two spatial dimensions, and determine the Lyapunov rate and the butterfly velocity in an extended random-phase approximation. The thermal diffusivity is found to be universally related to these chaos parameters *i.e.* the relationship is independent of N, the gauge coupling constant, the Fermi velocity, the Fermi surface curvature, and high energy details.

str-el] 12 Mar 2017

$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$v_B \sim \frac{N v_F^{5/3}}{e^{4/3}\gamma^{1/3}}\, T^{1/3}$$

$$D_E = 0.42 v_B^2 \tau_L$$

Patel & Sachdev

- Connections of the form $D \sim v_B^2 \tau_L$ have also been seen in

Electron/phonon bad metals - Werman, Kivelson & Berg
Incoherent Bose-Hubbard models - Bohrdt, Endrel, Mendes & Knap
O(N) models - Chowdury & Swingle
Fermi liquids with disorder/electron-electron interactions - Aleiner, Faoro & Ioffe
Higher derivative holographic theories - Baggioli, Gouteraux, Kiritsis & Li

- Recent work by Hartnoll et al provides motivation for a relationship between light-cone velocity and diffusion (see Sean's talk!)

Thank you!