Semi-quantum liquids

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Interparticle interaction can be characterized by a parameter

$$r_s = \frac{E_{\text{pot}}}{E_{\text{kin}}}$$

\[ E_{\text{kin}} \propto n^{2/D} \quad E_{\text{pot}} \propto n^{g/D} \]

$$r_s \propto n^{\frac{g-2}{D}}$$

(e-e interaction energy is \( V(r) \sim 1/r^g \))

Electrons \((g=1)\) form Wigner crystals at \(T=0\) and small \(n\) when \(r_s >> 1\) and \(E_{\text{pot}} >> E_{\text{kin}}\)

In the opposite limit the system is in the Fermi liquid state.

\(^3\)He and \(^4\)He \((g>2)\) are crystals at large \(n\)

At \(T=0\) the crystal melts by quantum fluctuations at \(r_s = r_c >> 1\)

Most quantum liquids in nature are strongly correlated with \(r_s > 1\).
Quantum liquid (Fermi or Bose)
Hierarchy of characteristic energies in weakly interacting liquids $r_s << 1$

**Fermi liquid**

- Weakly interacting Fermi liquid
- $E_{pot}$
- $\Theta$
- $E_d = E_F$

**Bose liquid**

- Weakly interacting superfluid liquid
- $E_{pot}$
- $\Theta$
- $E_d = T_c$

$\Theta$ is Debye (or plasma) frequency, $T_c$ is the Bose condensation critical temperature, $E_F$ is the Fermi energy
The subject of the talk:

If $E_d \ll T \ll \Theta \ll E_{pot}$ the liquid is not degenerate. It is still not a gas! It is also not a classical liquid!
A digression: features of classical liquids:

Classical liquids exist because crystal melting temperature is about 100 times less than $E_{pot}$.

They exhibit short range order and, in this sense, they are close to crystals. (Frenkel 1946, Shockley 1949)

a model:

Particles quickly oscillate in a cage made by other particles with the frequency $\Theta$.

Lifetime in the cage: $\tau >> 1/\Theta$.

$$\frac{1}{\tau} \approx \Theta \exp\left( - \frac{E_{pot}}{T} \right)$$

$\eta \propto \tau$ is the liquid's viscosity.
Viscosity of classical liquids decreases exponentially with temperature. Viscosity of gases increases with temperature.

What is the viscosity of semi-quantum liquids?

What is the heat capacity of semi-quantum liquids?

What is the resistivity of conductors in semi-quantum regime?
The effective mass and the spin susceptibility of Fermi liquids are significantly enhanced.

Bose-condensed fraction in Bose liquids is small.

\[ E_d \approx \frac{\hbar}{\tau}; \quad m^* \approx \frac{\hbar \tau}{a^2} \]

\( a \) is an interparticle distance

A model for strongly correlated degenerate liquids: particles quickly oscillate in a cage made by other particles with frequency \( \Theta \). The life time in the cage \( \tau >> 1/E_F \).
Snapshot of the single particle potential in semi-quantum liquids. On the time scale $t \ll \tau$ the system looks like a glass!
The low energy ($\epsilon<\Theta$) density of states of glasses $\nu$ is temperature independent. (Halperin, Anderson, Varma)

A conclusion: the heat capacity of semi-quantum liquids at $T<\Theta$ is proportional to $T$, independently of the particle’s statistics (Bose or Fermi)

$$C \approx \nu T$$

What about experiment?

$$\Theta^{He^4} \approx 40 K \quad \Theta^{He^3} \approx 20 K$$

Temperature dependence of the entropy per atom: $\Theta^{He^4}$ at saturated vapor pressure, $\bullet$—$He^4$ at 25 atm, $+$—$He^4$ at 25 atm, $\times$—parahydrogen at pressure 64.6 atm.$^{10}$

FIG. 1. Temperature dependence of the entropy per atom: $\Theta^{He^4}$ at saturated vapor pressure, $\bullet$—$He^4$ at 25 atm, $+$—$He^4$ at 25 atm, $\times$—parahydrogen at pressure 64.6 atm.$^{10}$

Temperature dependence of the heat capacity of $He^3$ and $He^4$ in the temperature interval 1-20K is not known. WOW!
Steps to calculate the viscosity of semi-quantum liquids:
1. Assume: there is no characteristic frequencies in the system smaller than \( E_d \sim 1/\tau \)
2. Apply shear deformation \( e(\omega) \). Calculate the energy dissipation
3. Put \( \omega = 1/\tau \)

\[
\eta(\omega) = \eta(\omega = 0) f(\omega \tau)
\]

\[
\frac{dE}{dt} = \frac{1}{2} \omega^2 e^2(\omega) \eta(\omega); \quad T \gg \omega \gg E_d = \frac{1}{\tau}
\]

\[
\frac{dE}{dt} = \frac{\pi \hbar \omega}{2} T \frac{\omega}{\hbar} \left| V \right|^2 \left| e \right|^2 n \nu \approx \frac{\pi \omega^2}{T} \left| V \right|^2 \left| e \right|^2 n \nu
\]

\[
\eta(\omega = 0) \propto \frac{\hbar}{T} n \nu \left| V \right|^2 = \hbar S \frac{\left| V \right|^2}{T^2}
\]
Temperature dependence of the viscosity of He\textsuperscript{3} and He\textsuperscript{4} in the temperature interval 1-20K is not known. WOW!

\[ E_D < T < \Theta \]

\[ \eta \propto \frac{1}{T} \]
$$\eta(\omega = 0) \propto \frac{\hbar}{T} n v |V|^2 = \hbar S \times \frac{|V|^2}{T^2}$$

S is the entropy density

**“Minimal” viscosity:**

Boltzmann transport result:

$$\eta \propto \rho v l \propto \hbar S \frac{l}{\lambda}$$

$$l = \lambda \Rightarrow \eta \approx \hbar S$$
Does the semi-quantum regime exist in the electron system in semiconducting quantum wells? In semi-quantum liquids hydrodynamics works starting with the spatial scale of order inter-electron spacing

The case of rare strong impurities

Stokes law:

\[ F \propto u \eta \frac{1}{\ln(\eta / au)} \]

\[ D = 2 \]

\[ \rho \propto \eta \propto \frac{1}{T} \]

The case of smooth potential:
Andreev, Kivelson, Spivak

\[ \rho \propto a \eta + b \frac{1}{\kappa} \propto \frac{1}{T} \]
FIG. 2 Non-monotonic temperature dependence of the resistivity in a (100) Si MOSFET (a), p-GaAs quantum well (b), p-SiGe quantum well (c), and (111) Si MOSFET (d) deep in the metallic regime over an extended temperature range. The bare (non-renormalized) Fermi temperatures are 7.5 K (a), 0.75 K (b), and 7 K (c). Adapted from Mokashi and Kravchenko (2009) (a), Gao et al. (2005) (b), and Coleridge (1997) (c). Panel (d) courtesy of R. N. McFarland and B. E. Kane.
2D holes in a 10nm GaAs quantum well resistivity vs. $T$ at $B=0$

hole density $p = 1.3 \times 10^{10}/\text{cm}^2$

Frequency dependence of resistivity is unavailable
Linear temperature dependence of conductivity in Si two-dimensional electrons near the apparent metal-to-insulator transition

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Steps to calculate of conductivity of semi-quantum electronic liquids:

1. Assume: no characteristic frequencies in the system smaller than $E_d \sim 1/\tau$
2. Calculate energy dissipation in the presence of electric field $E(\omega)$
3. Put $\omega = 1/\tau$

$$
\frac{dE}{dt} = E^2(\omega)\sigma(\omega); \quad T \gg \omega \gg E_d = \frac{1}{\tau}
$$

$$
\sigma(\omega) = \sigma(\omega = 0) f(\omega \tau)
$$

$$
\frac{dE}{dt} = \frac{\pi \hbar \omega}{2} n \nu |\mathbf{V}|^2 |eEa|^2 n \nu \approx \frac{\pi \hbar \omega^2}{T} |\mathbf{V}|^2 |eEa|^2 n \nu
$$

$$
\sigma(\omega = 0) \propto \frac{\hbar (ea)^2}{T \tau^2} |\mathbf{V}|^2 n \nu;
$$

$\rho \propto T$!
Linear in $T$ resistivity of strongly correlated metals at high temperatures

$\text{Sr}_2\text{RuO}_4$

$\rho = \rho_0 (1 + \alpha (T - T_c))$

$\rho [\text{n}\Omega \cdot \text{m}] = 15.4(1 + 0.00451 (T[K] - T_c[K])$

Tyler & Mackenzie, Physica C (97)
Drude conductivity: \( \sigma_D = \frac{e^2 \nu_F l \nu}{3} \)

Yoffe –Regel limit

\( l \approx \lambda_F \approx a \Rightarrow \sigma_D \approx \sigma_M \approx \frac{e^2}{ah} \)

\( \sigma_M \) is the Mott's minimal conductivity

How small could the conductivity be in the framework of the semi-quantum liquid theory?

\[
\sigma_{sq}(T = \Theta) = \sigma_M \frac{E_F^*}{\Theta} \left( \frac{E_F^* \nu}{\nu} \right) \left| \tilde{V} \right|^2 < \sigma_M
\]
Drude conductivity based on conventional Fermi liquid theory decreases with frequency

\[ \sigma(\omega) = \frac{\sigma_D}{1 + (\omega\tau_{tr})^2} \]

In the framework of the semi-quantum liquid the conductivity increases with the frequency
La\textsubscript{1.9} Sr\textsubscript{0.1} Cu\textsubscript{O}\textsubscript{4} at specified temperatures. The inset shows the in-plane resistivity data up to 1000 K. From Hussey et al., 2004.
Conclusion:

Semi-quantum interval of temperature in strongly correlated liquids, probably, exists. Almost nothing is known about it either experimentally or theoretically.
The Pomeranchuk effect.

The semi-quantum regime.

The Fermi liquid regime.

The liquid He$^3$ is also strongly correlated liquid: $r_s; \ m^*/m >>1$. 

The temperature dependence of the heat capacity of He$^3$. 

He$^3$ phase diagram: