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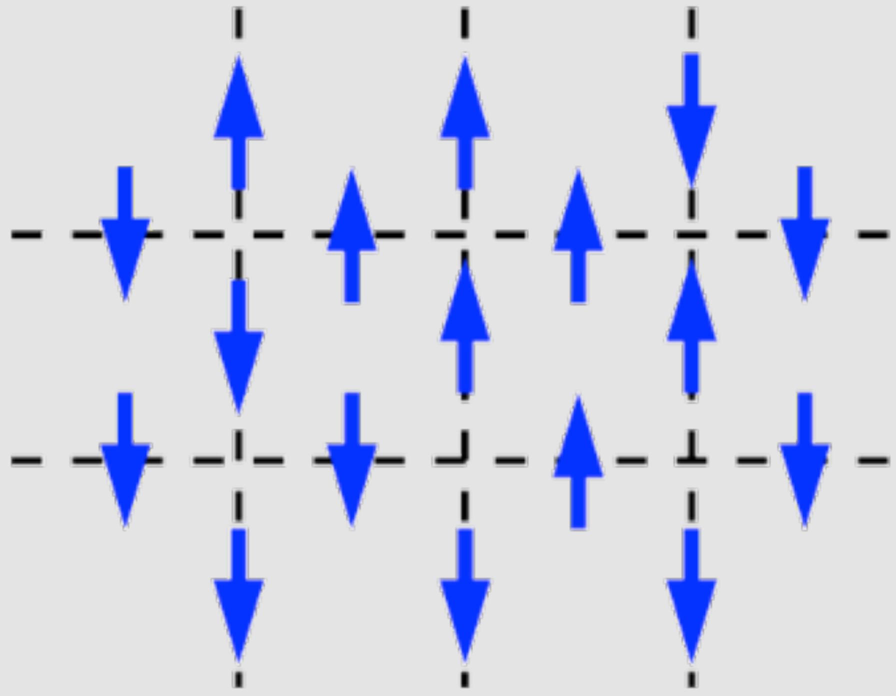

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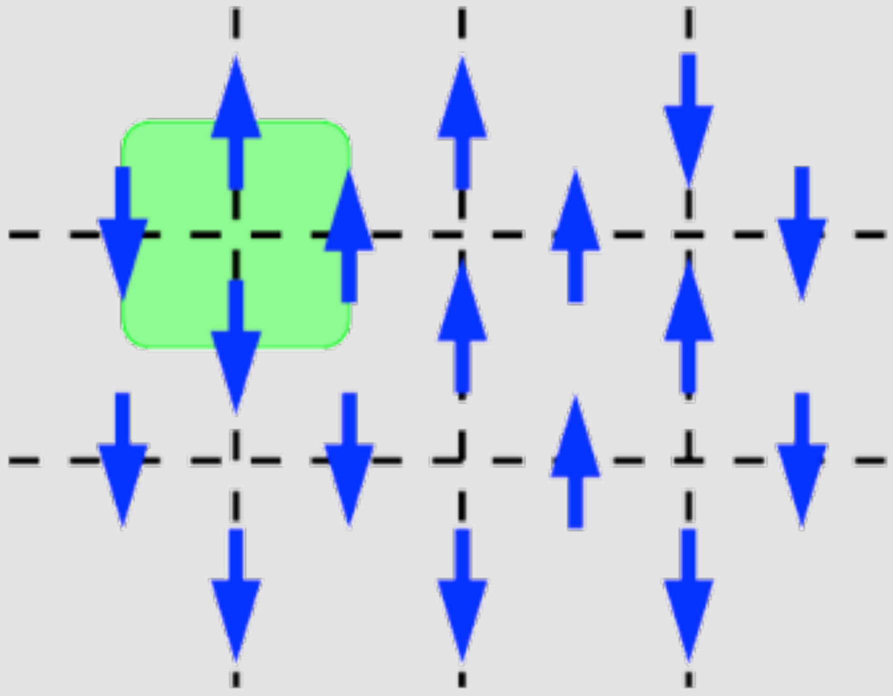

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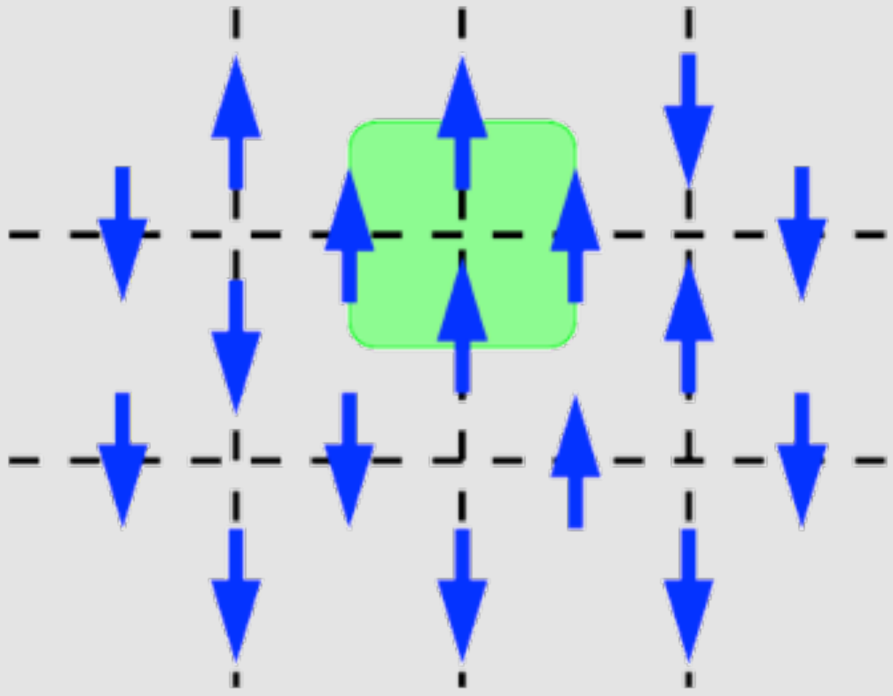

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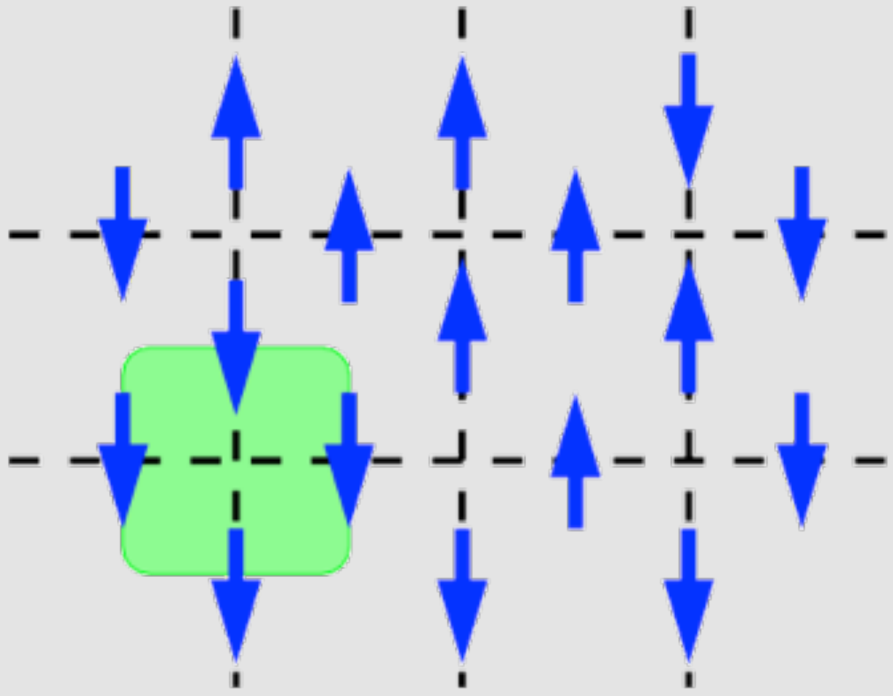

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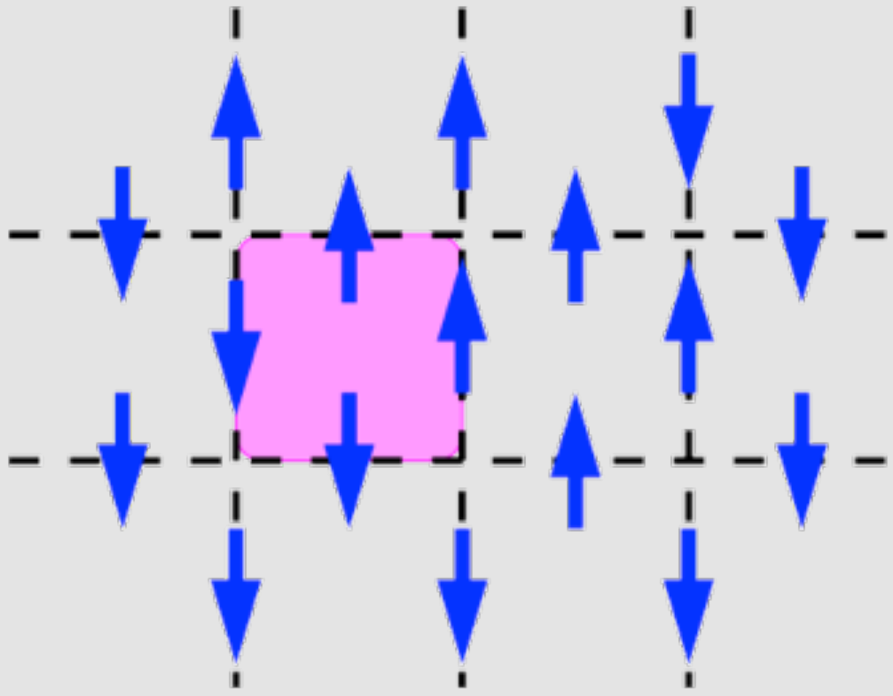

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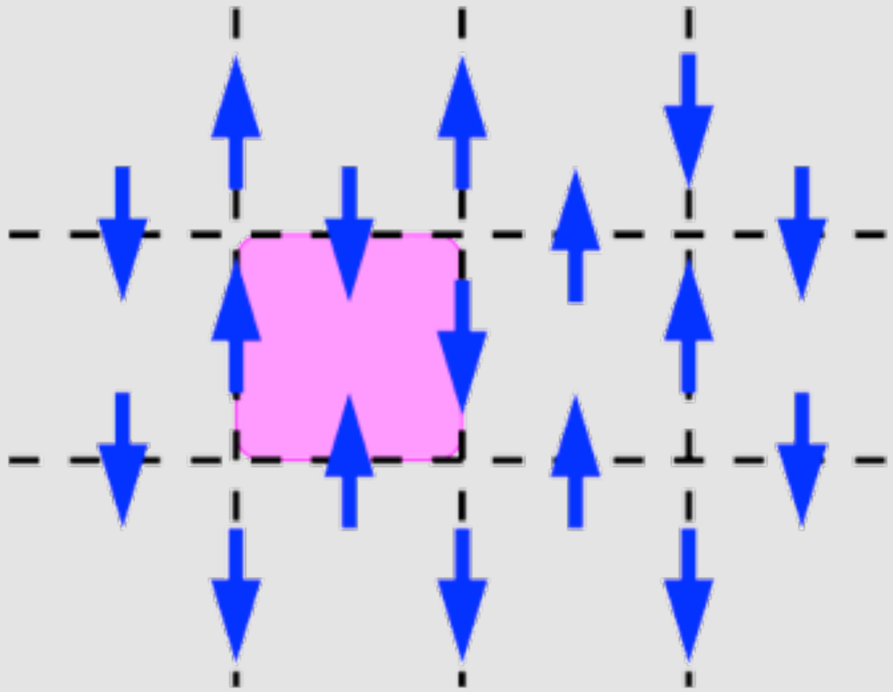

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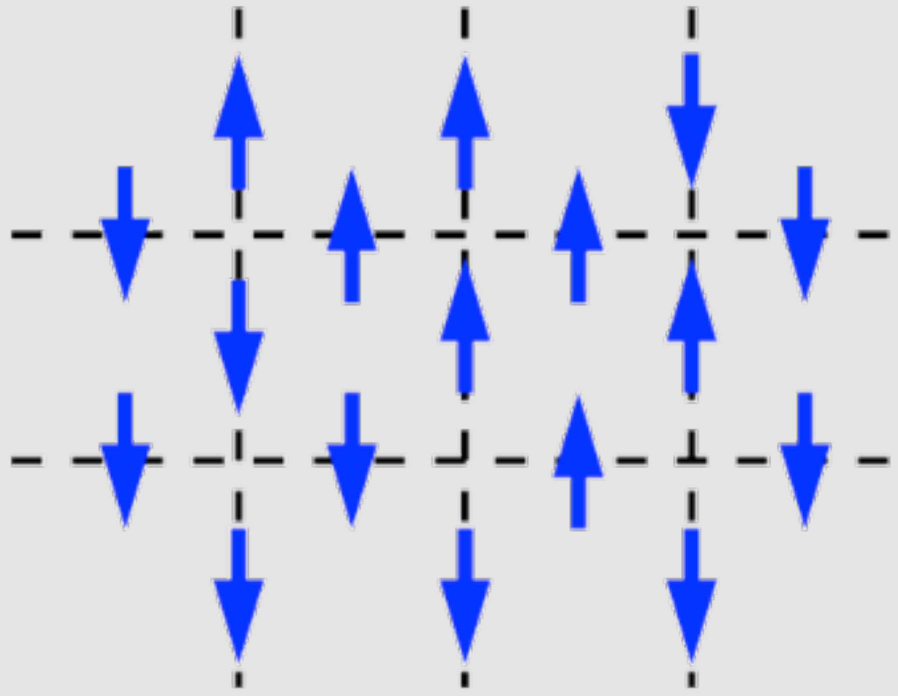

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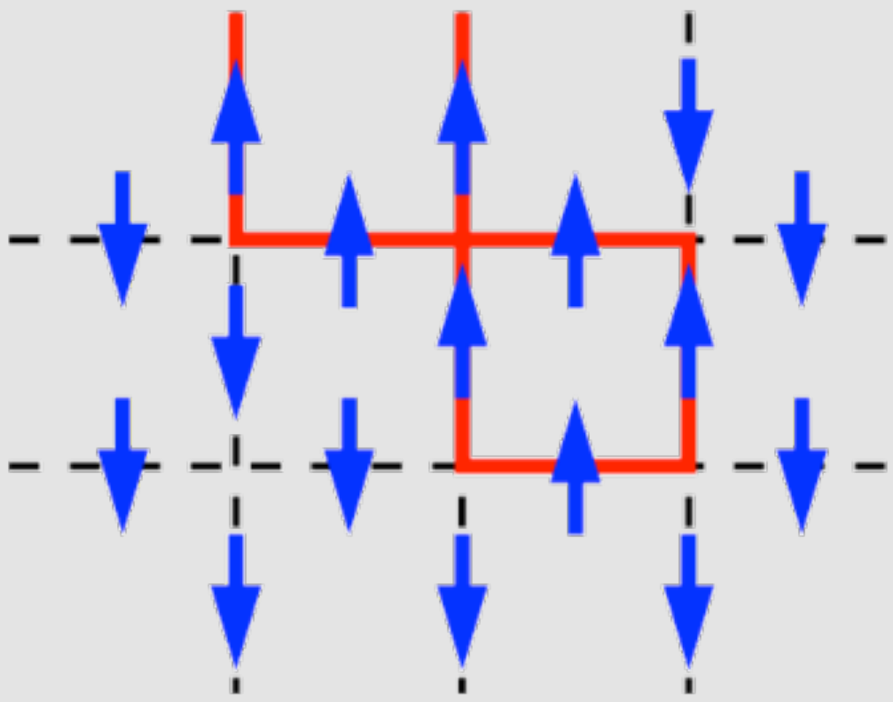

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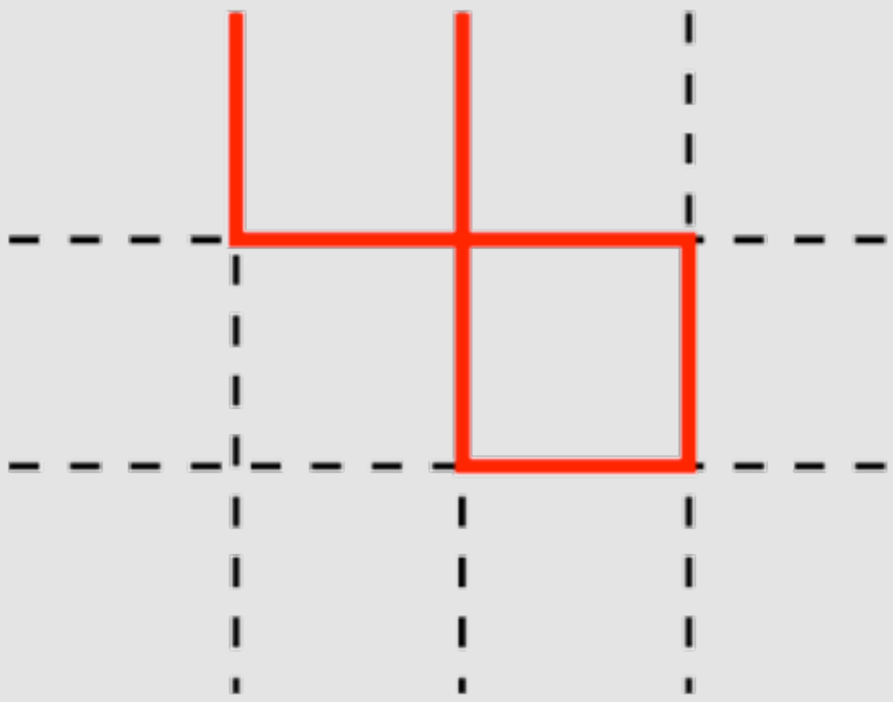

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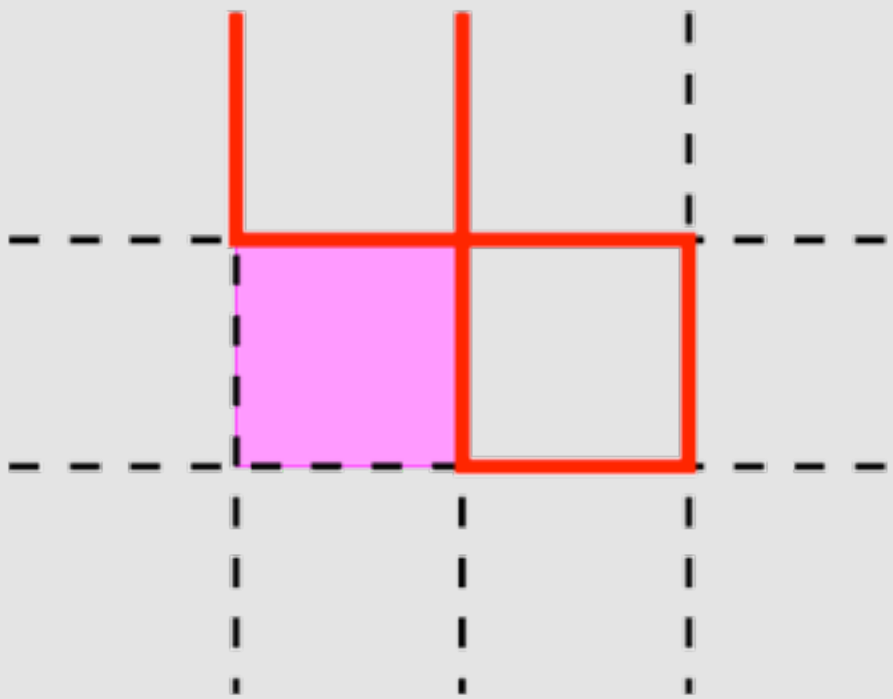

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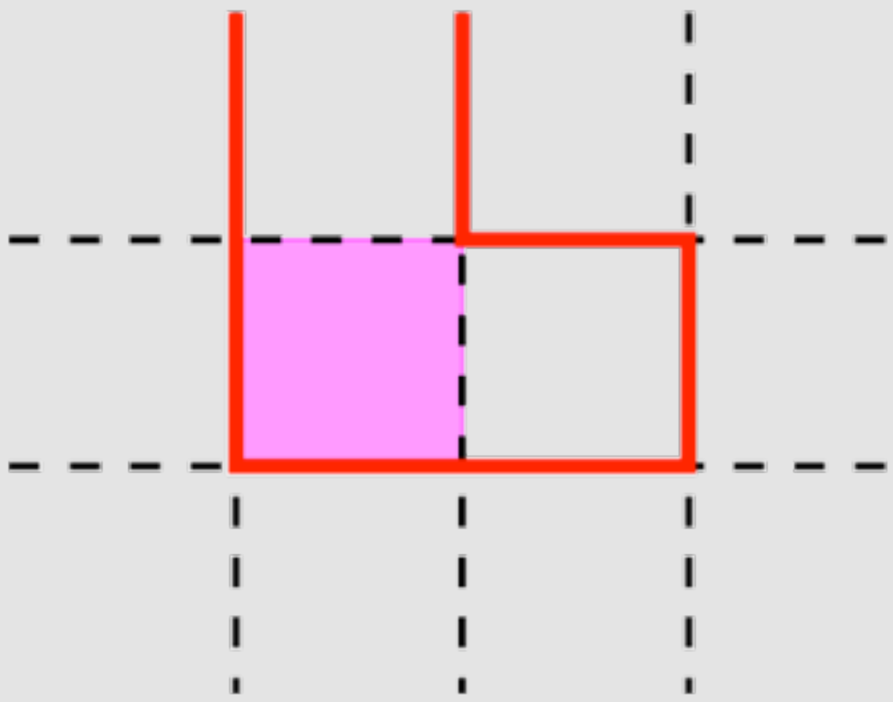

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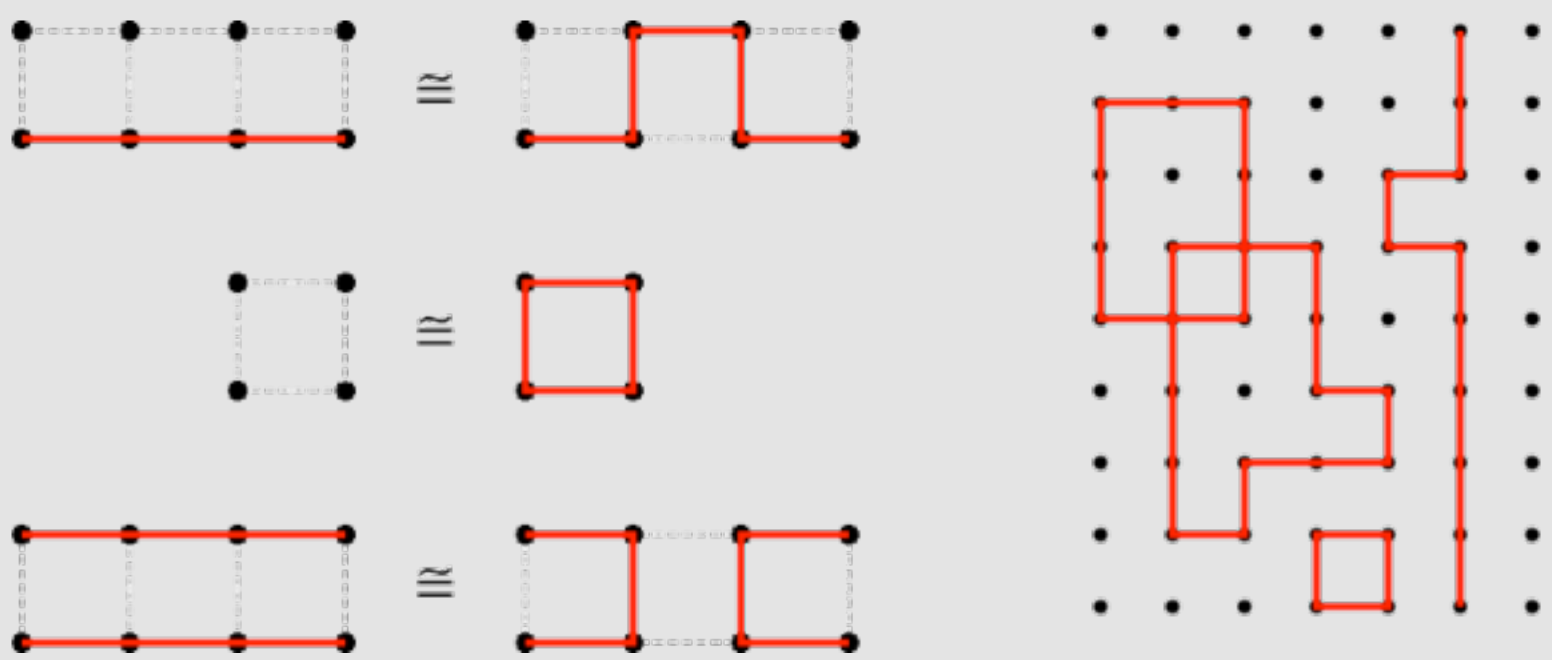
Properties of the Ground State

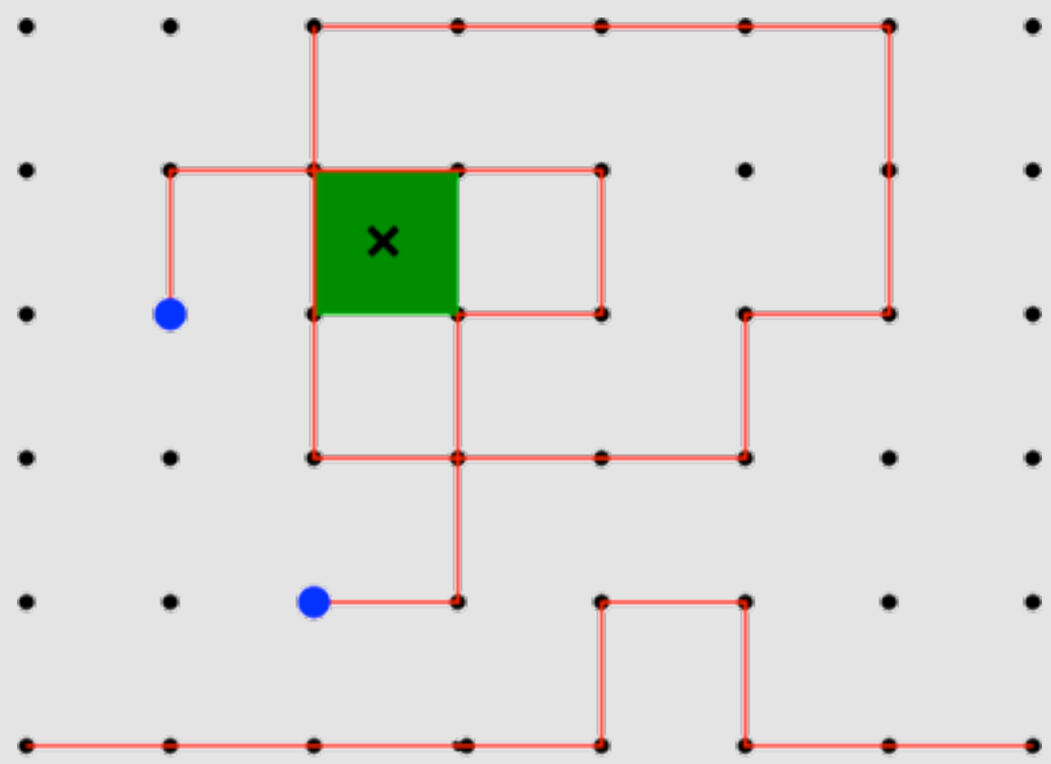
- Ground state: all $A_i=1$, $F_p=1$
- Pictorial representation: color each link with an up-spin.
- $A_i=1$: closed loops.
- $F_p=1$: every plaquette is an equal-amplitude superposition of inverse images.



The GS wavefunction takes the same value on configurations connected by these operations. It does not depend on the geometry of the configurations, only on their topology.

Properties of Excitations

- "Electric" particle, or $A_i = -1$ – endpoint of a line
- "Magnetic particle", or *vortex*: $F_p = -1$ – a "flip" of this plaquette changes the sign of a given term in the superposition.
- Charges and vortices interact via topological Aharonov-Bohm interactions.



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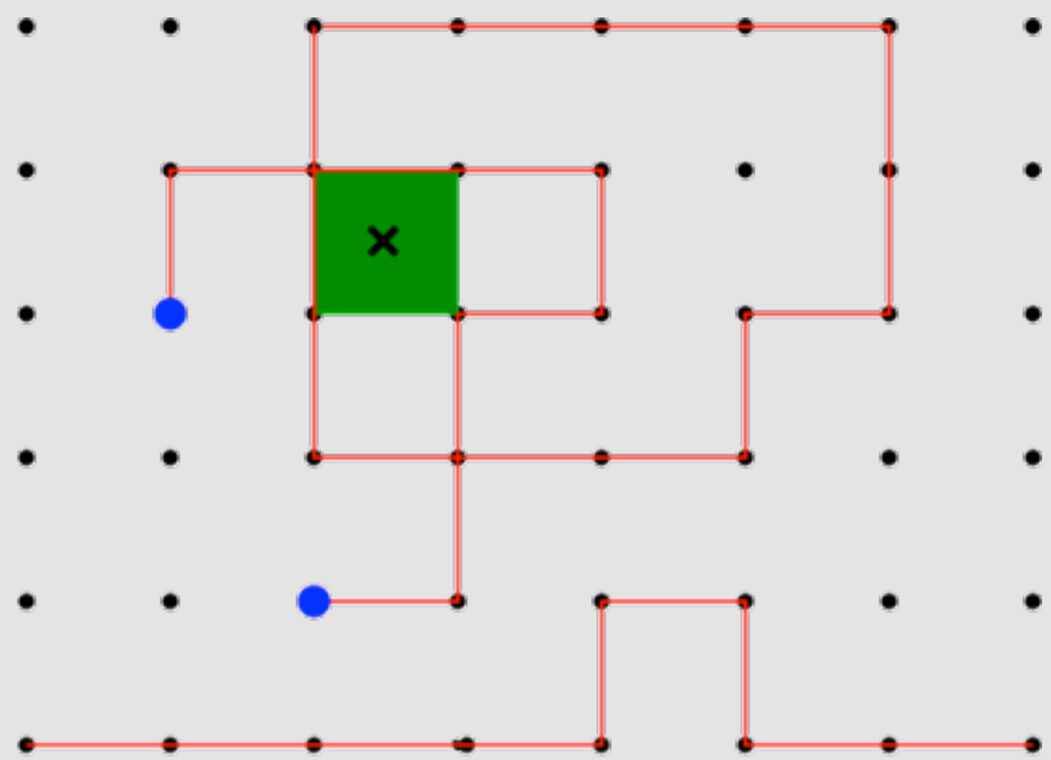

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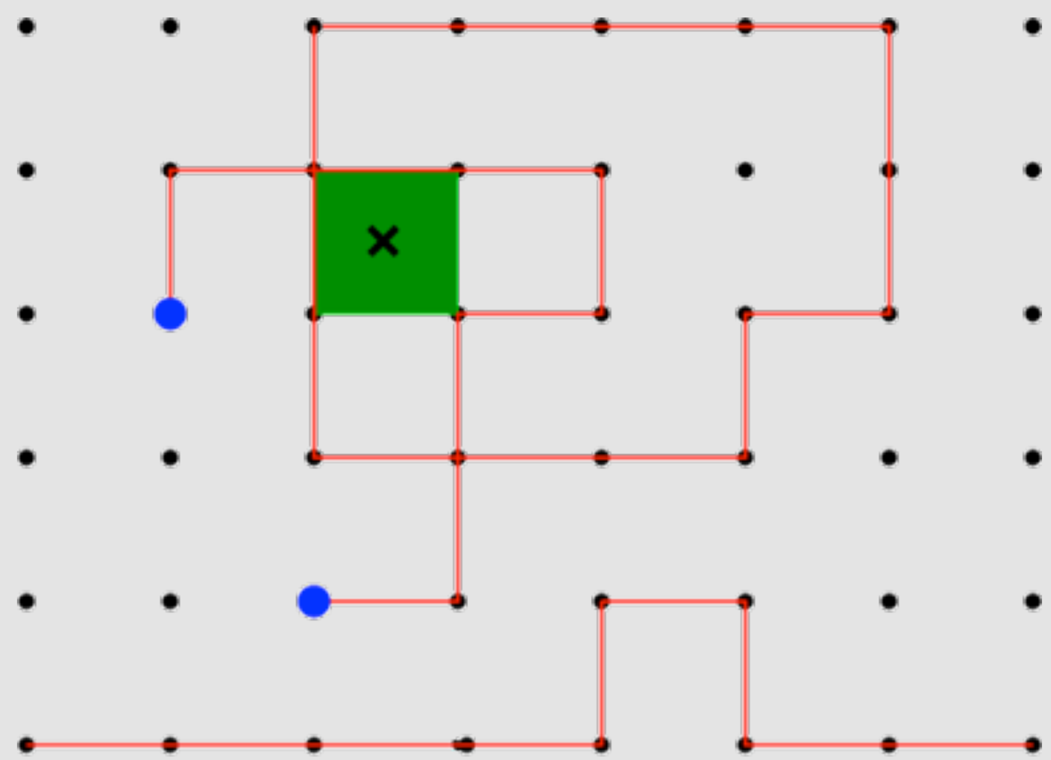

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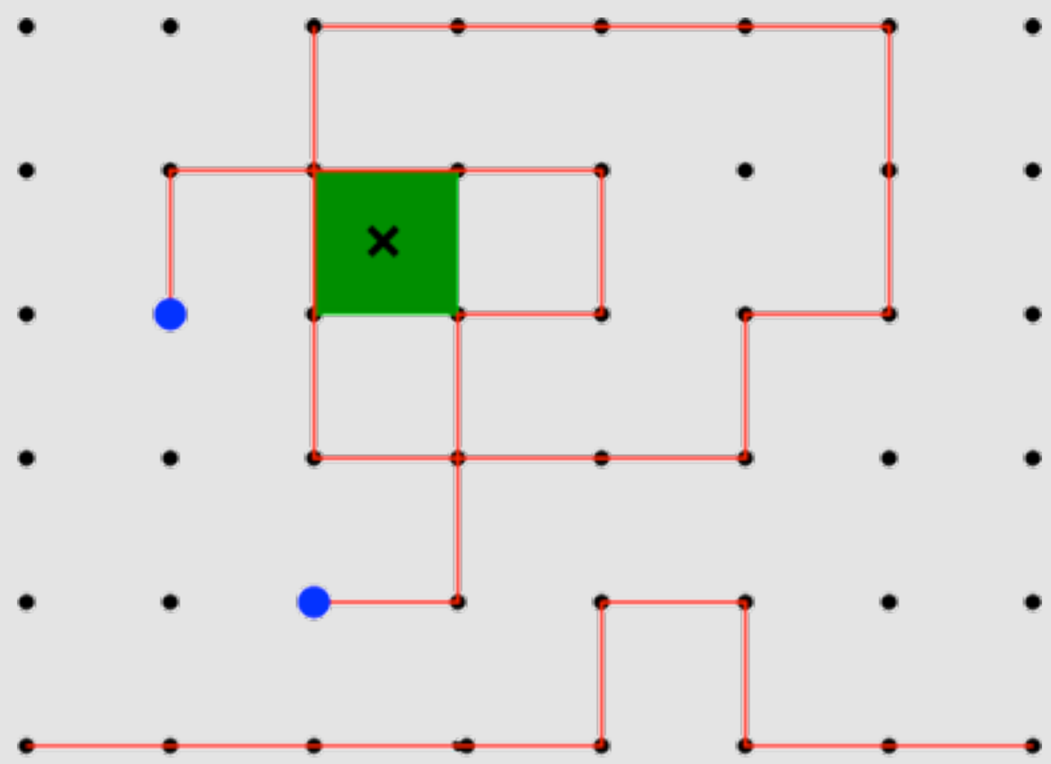

- *Spinons and visons are mutual semions*