Comment on "Spin Transport properties of the quantum one-dimensional non-linear sigma model"

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Abstract

In a recent preprint (cond-mat/9905415), Fujimoto has used the Bethe ansatz to compute the finite temperature, zero frequency Drude weight of spin transport in the quantum O(3) non-linear sigma model in a magnetic field $H \neq 0$. We show here that, contrary to his claims, the results are in accord with earlier semiclassical results (Sachdev and Damle, Phys. Rev. Lett. **78**, 943 (1997)). We also comment on his $1/N$ expansion, and show that it does not properly describe the long-time correlations.

In a recent preprint [1], Fujimoto has considered non-zero temperature (T) transport in the one-dimensional quantum O(3) non-linear sigma model. He considers the frequency (ω) dependent spin-conductivity, $\sigma(\omega)$, and tests for the possibility that it has a term of the form

$$\mathrm{Re}\,\sigma(\omega) = K\delta(\omega) + \ldots. \tag{1}$$

In the presence of a non-zero magnetic field, $H \neq 0$, he uses a Bethe ansatz computation to show in the low-temperature limit that $K \sim \sqrt{T} e^{-(\Delta - H)/T}$, where Δ is the magnitude of the $T = 0$ energy gap.

Here we will show that, contrary to the claims of Fujimoto [1], this result is in precise accord with earlier semiclassical results [2]. For a classical system, the dynamical spin conductivity is given in terms of the the time (t) autocorrelation of the total spin current $J(t)$ as

$$\sigma(\omega) = \frac{1}{TL} \int_0^\infty \langle J(t) J(0) \rangle e^{i\omega t} dt, \tag{2}$$

where L is the size of the system, and, in the notation of Ref [2], the spin current is

$$J(t) = \sum_k m_k \frac{dx_k(t)}{dt}, \tag{3}$$

where m_k are the azimuthal spins of classical particles on trajectories $x_k(t)$. Then the average over spins given in Eqn 3 of Ref [2] shows that

$$\langle J(t) J(0) \rangle = A_1 \sum_{k,\ell} \left\langle \frac{dx_k(t)}{dt} \frac{dx_\ell(0)}{dt} \right\rangle + A_2 \sum_k \left\langle \frac{dx_k(t)}{dt} \frac{dx_k(0)}{dt} \right\rangle. \tag{4}$$

We will now show that the first term proportional to A_1 above contributes only to K; the second term proportional to A_2 yields only regular diffusive contributions to $\sigma(\omega)$, and these latter terms were the focus of attention in Ref [2]. The terms proportional to A_1 were also discussed in Ref [2], but Fujimoto appears to have overlooked them. In the semiclassical model, the set of velocities at time t is simply a permutation of the velocities at $t = 0$, and so in the first summation in (4) we can relabel the particles at time t such that $dx_{k=\ell}(t)/dt = dx_\ell(0)/dt$. Then the average in the first term in (4) easily evaluates to an average over a single Maxwell-Boltzmann distribution, and we get

$$\langle J(t) J(0) \rangle = A_1 \frac{L\rho c^2 T}{\Delta} + A_2(\ldots), \tag{5}$$

where c is the velocity of 'light' in the sigma model, and ρ is the total density of particles. Combining (1), (2) and (5), and using expressions in Ref [2], we have

$$K = \sqrt{\frac{\pi T c^2}{2\Delta}} e^{-(\Delta - H)/T} \left(\frac{1 - 2e^{-2H/T} + e^{-4H/T}}{1 + e^{-H/T} + e^{-2H/T}} \right). \tag{6}$$

This result is valid for $H, T \ll \Delta$, but H/T arbitrary. In the low T limit at fixed $H \neq 0$ ($T \ll H \ll \Delta$), (6) agrees precisely with Fujimoto's result for K.

It is interesting that K vanishes as $H \to 0$ for fixed $T \ll \Delta$, and then the conductivity only has the diffusive contribution proportional to A_2 [2]. Fujimoto has only quoted results in the low temperature limit for fixed $H \neq 0$, and it would be interesting to extend his computations to $H = 0$ to access the complementary regime discussed in Ref [2]. Strictly speaking, a purely semiclassical method cannot rule out the possibility that neglected quantum interference effects in special integrable systems will lead to a small non-zero K at $H = 0$, but we can expect that K should at least be suppressed by factors of order (thermal de Broglie wavelength)/(spacing between particles) from its nominal $H \neq 0$ values. Purely diffusive transport is possible only at $H = 0$, and more generally in models with strict particle-hole symmetry [2–4]. It is interesting that a similar phenomenon has been noted in the interacting electron models by Zotos *et al* [5], who were able to prove ballistic transport only in models without particle-hole symmetry.

Next, we comment on the $1/N$ expansion of transport properties. Any kind of bare $1/N$ expansion [1], or even the solution of a $1/N$-derived quantum Boltzmann equation [6], is doomed to failure at low T due to non-perturbative effects special to one spatial dimension. Transport involves collisions of particles, and at low T two-particle collisions dominate. The exact S-matrix [7] for such collisions is known at general N — it is $\mathcal{S}_{m_1' m_2'}^{m_1 m_2}(\theta)$ where θ is a rapidity difference, and particles with spins m_1, m_2 scatter into particles with spins m_1', m_2'. Now for large N, at fixed θ, we have

$$\mathcal{S}_{m_1' m_2'}^{m_1 m_2}(\theta) = \delta_{m_1 m_1'} \delta_{m_2 m_2'} + \mathcal{O}(1/N) \tag{7}$$

which corresponds to ballistic *transmission of spin*, along with a small amount of scattering at order $1/N$. However at low T, small rapidities dominate, and we should really take the limit $\theta \to 0$ at fixed N. In this case we find, for any fixed N

$$\lim_{\theta \to 0} \mathcal{S}_{m_1' m_2'}^{m_1 m_2}(\theta) = (-1) \delta_{m_1 m_2'} \delta_{m_2 m_1'}. \tag{8}$$

This corresponds to *total reflection of spin*, and was the key effect behind the diffusive behavior discovered in Ref [2]. This effect will not be captured at any finite order in the $1/N$ expansion; this makes all conclusions drawn from the $1/N$ expansion in Ref [1] unreliable.

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