Comment on "Universal Magnetic Properties of $La_{2-\delta}Sr_{\delta}CuO_4$ at Intermediate Temperatures"

In a recent Letter [1] Chubukov and Sachdev analyzed the universal behavior associated with the zero temperature critical point of the (2+1)-dimensional quantum Heisenberg model. This critical point separates a magnetically ordered phase from a singlet phase with a gap to all spin excitations. It is convenient to consider tuning the critical behavior by varying a parameter, g, through a critical value, g_c, with $g < g_c$ implying order and $g > g_c$ disorder. Chubukov and Sachdev further argued that their calculations agree quantitatively with data on pure and lightly doped $La_{2-\delta}Sr_{\delta}CuO_4$, implying that the principal effect of doping La_2CuO_4 is to tune the parameter g from $g < g_c$ to $g > g_c$. In this Comment I argue that the doping dependence of the magnetic susceptibility differs in an important way from the predictions of [1], so that the effect of doping is not simply to tune the system through the quantum phase transition.

Figure 1 shows experimental and theoretical spin susceptibilities. The straight solid line labeled $g = g_c$ is the magnetic susceptibility, in physical units, predicted by the theory of Ref. [1] at $g = g_c$ with the spin-wave velocity set to 0.7 eV Å as appropriate for La_2CuO_4. The curve labeled $g > g_c$ shows a susceptibility in the disordered phase. The dotted curve labeled $\delta = 0$ is the measured susceptibility for La_2CuO_4 [2], corrected for core and Van Vleck terms, as described in Ref. [3] and Fig. 1 caption. The measured susceptibility of La_2CuO_4 is consistent with the theoretical result [1] for $g < g_c$. The three-dimensional ordering of La_2CuO_4 causes deviations (not shown) from the ideal 2D Heisenberg behavior, which is shown as dotted lines. These results make it clear that tuning g from $g < g_c$ to $g > g_c$ reduces the magnitude and increases the slope of $\chi(T)$, at any temperature. The dotted and dashed lines in the figure are experimental data obtained from published data as described in the figure caption. Although there is some disagreement between the different data sets, two significant trends are clear. (1) At any fixed temperature, adding carriers *increases* χ_s, and, (2) at intermediate temperatures, adding carriers flattens the $\chi_s(T)$ curve. Further, $\lim_{T \to 0} \chi_s(T)$ seems to be nonzero.

I now consider the interpretation of these data. As T increases beyond 400 K, the $S = 1/2$ Heisenberg model and the observed χ in La_2CuO_4 cross over to the universal quantum critical regime described in Ref. [1]. From the theory of Ref. [1] one would expect that in this regime adding carriers would increase g and therefore decrease χ_s and make it drop more steeply; in fact, the opposite happens. Therefore, I conclude that at the intermediate temperatures, where the theory of Ref. [1] should apply, the observed χ_s differs from the expected χ_s. Interestingly, at lower T ($T < 300$ K) the susceptibility does acquire

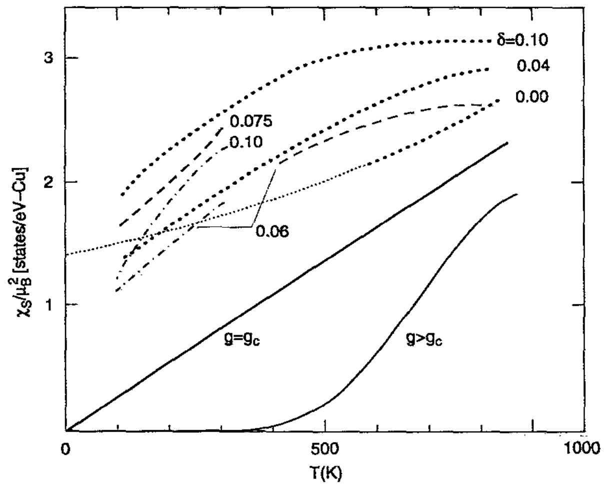

FIG. 1. Theoretical and experimental spin susceptibilities. The solid lines are theoretical results from Ref. [1]; the dashed and dotted lines are experimental data for $La_{2-\delta}Sr_{\delta}CuO_4$ for the δ values indicated. The heavy dotted lines: Ref. [2]; the dashed line: Ref. [4]; the dash-dotted line: Ref. [5]; and the long-dashed line: Ref. [6]. For $\delta = 0$ the lighter dots at low T represent the theoretical 2D Heisenberg model behavior. The spin susceptibilities were extracted from the measured bulk susceptibilities χ_m via $\chi_x = \chi_m - \chi_c - \chi_{VV}$. I used $\chi_c + \chi_{VV} = 1.0$ state/(eV Cu) for fields perpendicular to the CuO_2 plane and $\chi_c + \chi_{VV} = -1.25$ states/(eV Cu) for fields parallel to the CuO_2 plane. The numbers differ slightly from those given previously [3] because of misprints in both the original paper and the erratum. The powder average of $\chi_c + \chi_{VV}$ is -0.5 state/(eV Cu). For reference I note 1 state/(eV Cu) $= 33 \times 10^{-6}$ emu/mol Cu.

a more rapid T dependence, suggesting that a crossover may occur to a regime controlled by the fixed point discussed in Ref. [1].

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Chubukov and Sachdev Reply: In the preceding Comment [1], Millis has pointed out some interesting trends in the doping (δ) and temperature (T) dependence of the uniform susceptibility (χ_u) of $La_{2-\delta}Sr_\delta CuO_4$. However, his comparison of these data with the coupling constant (g) and T dependence of χ_u in the O(3) nonlinear sigma (NLσ) model is not appropriate, and is inconsistent with our analysis of the relationship between doped antiferromagnets and the NLσ model [2]. We have argued that a $T=0$ transition from a Néel state to a commensurate, quantum-disordered state in a lightly doped antiferromagnet should have the same leading critical behavior as the corresponding transition in the NLσ model [2]. However, we have also noted that the corrections to scaling in doped antiferromagnets should be quite different from those in the NLσ model [2]. Below, we discuss such corrections in the Shraiman-Siggia [3] model of doped antiferromagnets, and show that they cannot be neglected in χ_u in the quantum-disordered phase. We argue that the corrected results agree with most of the noted experimental trends [1].

The relevant terms in the Shraiman-Siggia model are

$$S=\int d\tau\, d^2r\left[\frac{(\partial_\tau n_l)^2+c^2(\nabla n_l)^2}{2g}+\Psi^\dagger_{v\alpha}(\partial_\tau-K_v)\Psi_{v\alpha}+\lambda\int d\tau\, d^2r(\Psi^\dagger_{v\alpha}\sigma^l_{\alpha\beta}\Psi_{v\beta})\mathbf{G}_v\cdot(\epsilon_{lmp}n_m\nabla n_p)\right], \tag{1}$$

where $\Psi_{v\alpha}$ is a hole-fermion field with valley index $v=1,2$ and spin index $\alpha=1,2$, n_l is the three-component field of the NLσ model, σ^l are the Pauli matrices, and λ is a coupling constant. The fermion kinetic energy, K_v, has valleys around $\mathbf{G}_{1,2}=(\pi/2,\pm\pi/2)$.

A crucial property of S is the absence of a three-body term like $\Psi^\dagger_{v\alpha}\sigma^l_{\alpha\beta}\Psi_{v\beta}n_l$. Such a term is forbidden by the sublattice interchange symmetry [3] under which the Ψ fields are invariant, but the n fields change sign. This absence makes it possible for the quantum-disordered phase to have fully gapped n quanta and gapless fermions, as the n particles are robust towards decay into fermion particle-hole pairs.

Let us now examine the response of S to a uniform magnetic field H. The Zeeman term couples H to the conserved total spin associated with the global SU(2) symmetry of S. The form of S in finite H is then specified by requiring that S be invariant under τ-dependent SU(2) transformations, under which H transforms like the τ component of a non-Abelian SU(2) gauge field. In the spirit of Hertz [4], we integrate out the fermion fields and obtain the following H-dependent effective action for the n field in the vicinity of the quantum transition,

$$\tilde{S}=-\mathrm{Tr}\ln(\partial_\tau-K_v-\sigma^l H_l/2)+\int d^2x\, d\tau\frac{1}{2\tilde{g}}[(\partial_\tau n_l-i\epsilon_{lmp}H_m n_p)^2+\tilde{c}^2(\nabla n_l)^2]\,. \tag{2}$$

The main *long-wavelength* consequences of the fermions has been to yield modified couplings $\tilde{g}$ and $\tilde{c}$ for the n field—this is the reason for our claim that the doping-induced transition has the same leading universal behavior as in the undoped NLσ model. We also note that the two terms in $\tilde{S}$ are *separately invariant* under the SU(2) gauge symmetry—thus there can be no symmetry-related constraint between the couplings. The value of χ_u can now be obtained by taking H derivatives of $\tilde{S}$:

$$\chi_u(T)=\chi_P+\chi_\sigma(T)\,, \tag{3}$$

where χ_σ is the universal NLσ model contribution computed by us [2], and χ_P is the Pauli susceptibility arising from the first term in $\tilde{S}$; in 2D, χ_P increases sharply with δ away from $\delta=0$. We see therefore that, at the critical point and in the disordered phase, $\chi_u(T\to 0)$ is finite because of the χ_P contribution.

Inclusion of χ_P yields results consistent with the data at small δ—there is an additional background contribution to χ_u which appears at any finite δ, while the slope of $\chi_u(T)$ remains nearly the same as in the undoped case. Further, we expect the quantum-critical behavior in χ_σ to appear at lower T with increasing δ, a trend again consistent with the data. Note also that according to [1], the value of $\chi_u(T\to 0)$ at $\delta=0.04$ is smaller than at $\delta=0$, this is also consistent with our result that the NLσ model contribution $\chi_\sigma(T\to 0)$ decreases with doping.

The behavior in the larger-δ, metallic, incommensurate phase is probably more complicated, but remarkably the dotted line at $\delta=0.1$ in [1] also follows quantum-critical dispersion at intermediate T, indicating that the n-field gap is rather small at $\delta=0.1$. Note also that according to [5], the maximum in $\chi_u(T)$ also shifts to lower T with doping, thus narrowing somewhat the region of universal low-T behavior. Why this happens remains unexplained.

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