

## Universal Magnetic Properties of $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$ at Intermediate Temperatures

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We present the theory of two-dimensional, clean quantum antiferromagnets with a small, positive, zero temperature ( $T$ ) stiffness  $\rho_s$ , but with the ratio  $k_B T/\rho_s$  arbitrary. Universal scaling forms for the uniform susceptibility ( $\chi_u$ ), correlation length ( $\xi$ ), and NMR relaxation rate ( $1/T_1$ ) are proposed and computed in a  $1/N$  expansion and by Monte Carlo simulations. For large  $k_B T/\rho_s$ ,  $\chi_u(T)/T$  and  $T\xi(T)$  asymptote to universal values, while  $1/T_1(T)$  is nearly  $T$  independent. We find good quantitative agreement with experiments and some numerical studies on  $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$ .

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The last few years have seen extensive theoretical and experimental studies of two-dimensional quantum Heisenberg antiferromagnets, with particular attention to the antiferromagnetism in the cuprate compounds [1,2]. On the theoretical side, most notable has been the work of Chakravarty, Halperin, and Nelson [3], who focused mainly on the low temperature ( $T$ ) properties of systems with well established long-range Néel order at  $T = 0$ ; their most detailed results were in a regime in which the fully renormalized,  $T = 0$ , spin stiffness  $\rho_s$  was not too small, while the temperature satisfied  $k_B T \ll \rho_s$ . Under these conditions, the antiferromagnet could be treated as a classical system, with all effects of quantum fluctuations being absorbed into renormalization of the couplings. At low  $T$ , there has been good agreement between their results and experiments on  $\text{La}_2\text{CuO}_4$  [3]. However, the experimental results at higher  $T$  remain poorly understood—there are clear deviations from the classical behavior and it is expected that quantum fluctuations will play a more fundamental role. Besides, in the lightly doped cuprates,  $\rho_s$  is likely to be quite small, thus decreasing the  $T$  range over which the renormalized-classical (RC) behavior will hold. Finally, there are experimental realizations of frustrated two-dimensional Heisenberg antiferromagnets [4], which, in all likelihood, have a very small value of  $\rho_s$ .

Our understanding of the experiments would clearly be improved by precise theoretical predictions in low temperature regimes other than  $k_B T \ll \rho_s$ . To this end, we discuss here some universal properties of clean two-dimensional quantum Heisenberg antiferromagnets with nearest-neighbor exchange  $J$ , in which the stiffness  $\rho_s$  is “small,” but nonzero. We will study the physics when  $0 < \rho_s \ll J$ ,  $k_B T \ll J$ , but the ratio  $k_B T/\rho_s$  is allowed to be arbitrary. The system is then controlled by renormalization-group flows near the  $T = 0$  quantum fixed point separating the Néel-ordered and quantum-disordered phases. Our main new result will be that, in this regime, the absolute values of the entire long-wavelength, low-frequency, uniform and staggered spin susceptibilities are completely universal functions of just three thermodynamic parameters:  $\rho_s$ ,  $c$ , and the ordered staggered moment  $N_0$ . The universal functions depend

only on the symmetry of the order parameter, and sensitivity to all lattice-scale physics arises only through the values of  $\rho_s$ ,  $c$ , and  $N_0$ . For small  $k_B T/\rho_s$  (the RC region), the  $T$  dependence of our results is similar to those already obtained in Ref. [3]. For large  $k_B T/\rho_s$  [the quantum-critical (QC) region of Ref. [3]], most of our results are new. We will show that they are consistent with the available experimental [5–7] and some of the numerical [8–10] data on the uniform susceptibility, correlation length, and NMR relaxation rate for undoped and weakly doped  $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$ . We thus argue that the use of a small  $\rho_s$  point of view is not unreasonable even for the pure square lattice, spin-1/2, Heisenberg antiferromagnet; while ordered at  $T = 0$ , this system is evidently close to the point where long-range order vanishes.

Our results follow from some very general properties of the  $T = 0$  quantum fixed point separating the magnetic-ordered and quantum-disordered phases. These properties are expected to be valid in both undoped and doped antiferromagnets, though not in the presence of randomness [11,12]. They are as follows: (i) The fixed point is described by a continuum (2+1)-dimensional field theory which is Lorentz invariant, and the spin-wave velocity,  $c$ , remains nonsingular through the phase transition. (ii) At  $T = 0$ , on the magnetic-ordered side, there is a Josephson correlation length  $\xi_J$  which diverges at the quantum fixed point; near this fixed point  $\rho_s$  equals  $\hbar c \Theta / \xi_J$ , where  $\Theta$  is a universal number [13,14]. (iii) Turning on a small  $T$  places the critical field theory in a “slab” geometry which is infinite in the two spatial directions, but of finite length  $L_\tau = \hbar c / k_B T$ , in the imaginary time ( $\tau$ ) direction—its consequences therefore follow from finite-size scaling [3].

*Uniform susceptibility,  $\chi_u$ .*—We first consider the response of the antiferromagnet to a static, spatially uniform, external magnetic field (the extension to a field at finite wave vector  $k$  or frequency  $\omega$  will be omitted here for brevity). Such a field causes a uniform precession of all the spins, which can be removed by transforming to a rotating reference frame at the price of a twist in the boundary conditions along the  $\tau$  direction [15]. The response of the system to this twisted boundary condition defines a stiffness,  $\rho_\tau$ , which equals  $\chi_u$ . However, the fixed point is Lorentz invariant, and hence  $\chi_u$  has the

same scaling properties as  $\rho_s$ . Application of finite-size scaling [16] then yields the following  $T$  dependence for  $\chi_u$ ,

$$\chi_u(T) = \left(\frac{g\mu_B}{\hbar c}\right)^2 k_B T \Omega_Q(x), \quad x \equiv \frac{Nk_B T}{2\pi\rho_s}, \quad (1)$$

where  $g\mu_B/\hbar$  is the gyromagnetic ratio,  $N$  is the number of components of the order parameter, and  $\Omega_Q(x)$  is a *universal* function. Note  $x \propto \xi_J/L_\tau$ , the length ratio expected in finite-size scaling functions. We have computed  $\Omega_Q(x)$  in a  $1/N$  expansion for the  $O(N)$  nonlinear sigma model in 2+1 dimensions [17]. The  $O(3)$  model describes the low-energy dynamics of two-dimensional Heisenberg antiferromagnets on a square lattice. The antiferromagnet also carries Berry phases, not present in the  $\sigma$  model, but these have been argued to be irrelevant at the quantum fixed point [11]. At  $N = \infty$ , the scaling function  $\Omega_Q(x)$  can easily be calculated:

$$\Omega_Q^{N=\infty}(x) = \frac{1}{\pi x} + \frac{\sqrt{4 + e^{-2/x}}}{\pi e^{-1/x}} \operatorname{arcsinh}\left(\frac{e^{-1/x}}{2}\right). \quad (2)$$

Of particular interest is the behavior of  $\chi_u$  for large  $x$ . The function  $\Omega_Q^{N=\infty}(x)$  is analytic at  $x = \infty$ , and the general principles of finite-size scaling [16] suggest that this remains true at finite  $N$ . Thus we expect that  $\Omega_Q(x \rightarrow \infty) = \Omega_\infty + \Omega_1/x + \dots$  with  $\Omega_\infty, \Omega_1$  universal numbers. Combined with (1), this implies that a plot of  $\chi_u(T)$  vs  $T$  will be a straight line at large  $T/\rho_s$  with universal slope and intercept, whose values are related to  $\Omega_\infty$  and  $\Omega_1$ , respectively. At  $N = \infty$  we obtain from (2)  $\Omega_\infty = (\sqrt{5}/\pi) \ln[(\sqrt{5} + 1)/2] \approx 0.3425$  and  $\Omega_1 = 4\Omega_\infty/5$ . We have computed the first  $1/N$  correction to  $\Omega_\infty$  and indeed found that it is a universal, regularization-independent number. We obtained

$$\Omega_Q(x = \infty) \equiv \Omega_\infty = 0.3425(1 - 0.619/N + \dots). \quad (3)$$

We have also performed Monte Carlo simulations of a *classical*  $D = 3$  Heisenberg ferromagnet on a cubic lattice, whose phase transition is expected to be in the same universality class as the  $O(3)$  sigma model. We used a lattice of size  $L \times L \times L_\tau$  ( $L \leq 30, L_\tau \leq 10$ ) at its known critical coupling [18] and computed  $\rho_\tau$ . It then follows that  $\Omega_\infty = \lim_{L_\tau \rightarrow \infty} \lim_{L \rightarrow \infty} L_\tau \rho_\tau$ , where the order of limits is crucial. The result was  $\Omega_\infty = 0.25 \pm 0.04$ , in good agreement with the  $1/N$  result at  $N = 3$ . Finally, there is an analogy between  $\Omega_\infty$  and another universal number discussed recently—the universal conductivity,  $\sigma_Q$ , at the superfluid-insulator transition [14,19].

We turn next to small  $x$ . The  $N = \infty$  result (2) gives the leading term  $\Omega_Q(x \rightarrow 0) = 1/\pi x$ , which implies  $\chi_u(T \rightarrow 0) = 2g^2\mu_B^2\rho_s/\hbar^2 c^2 N \equiv (2/N)\chi_\perp$ , where  $\chi_\perp$  is the transverse susceptibility. This is in fact equal to the exact result expected from rotational averaging of an ordered quantum  $O(N)$  sigma model [3,20]—we have indeed found no corrections in the  $1/N$  expansion

at  $T = 0$ . Furthermore, it was shown in Ref. [3] that there are no  $T$ -dependent corrections to the isotropic  $\chi_u$  in a classical,  $d = 2$ , lattice rotor model. In contrast, for our quantum  $O(N)$  sigma model, the  $N = \infty$  result contains a term linear in  $T$  at small  $T$ . In the  $1/N$  expansion of this quantum model, the classical contributions to various observables appear as  $\ln x/N$  terms; however, as expected, all  $\ln x/N$  contributions to  $\chi_u$  were found to cancel among each other. We then calculated the regular  $1/N$  corrections and found

$$\Omega_Q(x \rightarrow 0) = 1/\pi x + \Gamma_N + \dots, \quad (4)$$

where  $\Gamma_N = (N - 2)/\pi N$  is a universal number.

*Correlation length,  $\xi$ .*—The scaling dimension of  $\xi$  is  $-1$ , and the finite-size scaling result for  $\xi(T)$  is therefore

$$\xi^{-1}(T) = (k_B T/\hbar c) X_Q(x), \quad (5)$$

where  $X_Q(x)$  is a universal function. As for  $\chi_u$ , there are no nonuniversal factors on the right-hand side. The numerical results for  $X_Q$  depend on the precise definition chosen for  $\xi$ : we follow Ref. [3] and define the correlation length from the long distance  $e^{-r/\xi}$  decay of the equal time order parameter correlation function. Equivalently, one can define  $\xi$  as  $\kappa^{-1}$ , where  $\kappa$  is the location of the pole of the staggered structure factor  $S(k)$  closest to the real  $k$  axis. At  $N = \infty$ , we have simply

$$X_Q^{N=\infty}(x) = 2\operatorname{arcsinh}[(1/2)e^{-1/x}]. \quad (6)$$

For large  $x$ , the properties of  $\xi^{-1}$  are similar to those of  $\chi_u$ . The function  $X_Q(x)$  is expected to be analytic at  $x = \infty$  with  $X_Q(x \rightarrow \infty) = X_\infty + X_1/x + \dots$ ; a plot of  $\xi^{-1}(T)$  vs  $T$  will be a straight line at large  $T$  with a universal slope and intercept, whose values are related to  $X_\infty$  and  $X_1$ , respectively. We have also computed the  $1/N$  correction to  $X_\infty$  and found  $X_\infty = 0.962(1 + 0.237/N)$ . From (6) it follows that  $X_1 = -0.894 + O(1/N)$ .

For small  $x$ , the  $N = \infty$  result (6) gives  $X_Q(x \rightarrow 0) = e^{-1/x}$ . However, unlike  $\chi_u$ , the RC spin fluctuations make a strong contribution to  $\xi$ , thus requiring careful consideration of the  $\ln x/N$  terms in the  $1/N$  expansion. We identified terms to order  $(\ln x/N)^2$ , exponentiated them, and found

$$X_Q(x \rightarrow 0) = Y_N x^{-1/(N-2)} e^{-N/(N-2)x}, \quad (7)$$

where  $Y_N = 1 + (3 \ln 2 - 1 + C)/N + O(1/N^2)$  is a universal number ( $C$  is the Euler constant,  $C \approx 0.5772$ ). The  $T$  dependence of this result agrees with that of Ref. [3]. The prefactor was also obtained by another method in Ref. [21].

Note that Ref. [3] obtained an interpolation formula for  $\xi$  in an  $\epsilon$  expansion which, when reexpressed in terms of  $x$ , becomes  $\xi \sim \operatorname{arcsinh}(\frac{c}{2} e^{-N/(N-2)x})$ . Our approach shows that the  $N/(N-2)$  factor in the exponent is present only at small  $x$ , where the perturbative series

is logarithmic.

*NMR relaxation rate,  $1/T_1$ .*—The relaxation of nuclear spins coupled to the antiferromagnetic order parameter (e.g., Cu nuclear spins in  $\text{La}_2\text{CuO}_4$ ) is given by  $1/T_1(T) = \lim_{\omega \rightarrow 0} 2\tilde{A}_\pi^2(k_B T/\hbar\omega) \int (d^2k/4\pi^2) \chi''(k, \omega)$ , where  $\chi''(k, \omega)$  is the imaginary part of the dynamic staggered susceptibility of the underlying quantum antiferromagnet, and  $\tilde{A}_\pi$  is the bare hyperfine coupling. This determines the scaling dimension of  $1/T_1$  at the quantum fixed point to be  $\eta$ , the critical exponent associated with spin correlations at criticality:  $\eta = 8/3\pi^2 N - 512/27\pi^4 N^2 + \dots$  in a  $1/N$  expansion [22] and the best current value at  $N = 3$  is  $\eta \approx 0.028$  [18]. The finite-size scaling form for  $1/T_1$  can be shown to be

$$1/T_1(T) = (2\tilde{A}_\pi^2 N_0^2/\rho_s) x^\eta R_Q(x), \quad (8)$$

where  $R_Q(x)$  is a completely universal function. Note the complete absence of nonuniversal normalization factors in (8).

We now consider the limiting behavior of  $R_Q(x)$  for large and small  $x$ . As before, at large  $x$ ,  $R_Q(x \rightarrow \infty) = R_\infty$ , a positive constant; the small value of  $\eta$  then implies that  $1/T_1$  is *essentially  $T$  independent* at high  $T$ . To leading order in  $1/N$ , we estimate  $R_\infty$  from the result for  $\chi''(k, \omega \leq T)$  in Ref. [11] to be  $R_\infty = 0.66/N$ . Note the factor of  $1/N - \chi''(k, \omega)$  is finite at  $\omega \rightarrow 0$  only due to the self-energy corrections. At small  $x$ , dynamical scaling [3,23] predicts that  $R_Q(x) \propto \xi(x)$ . The  $1/N$  expansion is again singular when  $x \ll 1$  and we will not discuss it here.

*Comparison with numerical and experimental results.*—We have so far presented general scaling forms for the magnetic properties of a two-dimensional quantum antiferromagnet which has  $\rho_s \ll J$ . Explicit scaling functions can be calculated at  $N = \infty$ , and examination of  $1/N$  corrections has been limited to those for  $\Omega_\infty$ . These corrections were, however, quite small, and we expect, in general, that  $1/N$  expansion is robust and numerically quite accurate for large values of  $x$ . On the other hand, at small  $x$ , the  $1/N$  expansion is logarithmically singular, and eventually changes the leading singularity in some of the scaling functions at  $x = 0$ ; the final low- $T$  behavior is the same as that in the RC scaling theory of Ref. [3]. The crossover between small and large  $x$  should occur for  $x$  around unity. Thus for  $x \geq 1$  (but such that the long-wavelength description is still valid), it is quite likely that  $1/N$  expansion will describe the experimental data better than the RC theory, which, strictly speaking, is valid only for  $x \ll 1$ . In a square lattice, nearest-neighbor,  $S = 1/2$  Heisenberg antiferromagnet,  $2\pi\rho_s \approx 1.13J$  (Ref [24]) and we therefore expect that our large- $N$ , large  $x$  results should work for  $x \geq 1$ , i.e., for  $k_B T \geq 0.35J$ .

The absence of any RC corrections to  $\chi_u$  makes it an ideal candidate for testing our theory; the  $1/N$  expansion should become accurate even at fairly small values of  $x$ . We start with the numerical results for

$\chi_u(T)$  on the square lattice  $S = 1/2$  antiferromagnet. There have been high- $T$  series expansions [8], quantum Monte Carlo calculations [9], and finite cluster calculations [10]. Their results all show that  $\chi_u(T)$  obeys a Curie-Weiss law at high  $T$ , reaches a maximum at  $k_B T \sim J$ , and then falls to a finite value at  $T = 0$ , which is rather close to the rotationally averaged  $1/S$  result  $(\hbar/g\mu_B)^2 \chi_u(T = 0) \approx 0.04/Ja^2$ , where  $a$  is the lattice spacing. For  $0.35J < T < 0.55J$ , both series expansions [8] and Monte Carlo [9] calculations report a *linear  $T$  dependence* of  $\chi_u(T)$  (Fig. 1). Also plotted is our theoretical prediction of Eqs. (2) and (3) which, over the range of  $x$  values used in the figure, is well approximated by  $(\hbar/g\mu_B)^2 [Ja^2 \chi_u(T)] \approx 0.037x(1 + \alpha/x)$ ,  $\alpha = 0.8 + O(1/N)$ . This is remarkably close to the best fit to the data of Ref. [9] which gives  $0.037x(1 + 0.775/x)$ . Moreover, the theoretical  $x$ -dependent term in the RC region is  $0.014x$ , in clear disagreement with the numerical data at  $T > 0.35J$ .

We consider next measurements of  $\chi_u(T)$  in weakly doped  $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$ . The interpretation of the experimental data even above the zero-doping  $T_N$  requires caution because one has to subtract Van-Vleck, core, and diamagnetic contributions from the measured  $\chi_u(T)$ . Nevertheless, after subtraction was carried out, it was found [5,25] that at small doping concentration the susceptibility is *linear* in  $T$ ; the slope of  $(\hbar/g\mu_B)^2 [Ja^2 \chi_u(T)]$  vs  $x$  is about 0.043, which is very close to our result (0.037). Thus for  $\text{La}_{1.95}\text{Sr}_{0.05}\text{CuO}_4$ ,  $\chi_u$  scales with  $T$  in the temperature interval  $100 \pm 400$  K (note that for 5% doping,  $\rho_s$  is already very small and the QC region is shifted to lower  $T$ ).

Now the  $^{63}\text{Cu}$  spin-lattice relaxation rate,  $1/T_1$ , in  $\text{La}_2\text{CuO}_4$ . At low  $T$  ( $x$  small), the theory of Ref. [23] predicts that  $1/T_1 \propto x^{3/2}e^{-3/x}$ ; this is in very good agree-

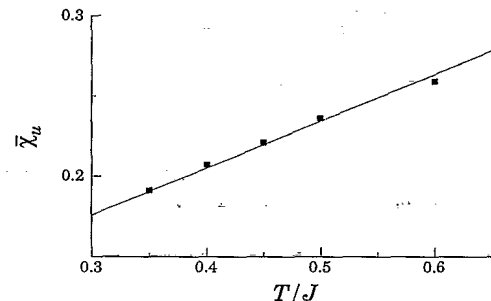


FIG. 1. Monte Carlo [9] (squares) and theoretical (line) results for the uniform susceptibility  $\bar{\chi}_u = [3J(a\hbar/g\mu_B)^2] \chi_u$  of a square lattice spin-1/2 Heisenberg antiferromagnet ( $a$  is the lattice spacing). The experimental results for  $\text{La}_{1.95}\text{Sr}_{0.05}\text{CuO}_4$  are very close to the Monte Carlo data [5]. There are *no* adjustable parameters in the theoretical result (1). Over the range of  $x$  plotted, the function  $\Omega_Q(x)$  is very close to its large- $x$  behavior  $\Omega_Q(x) \approx \Omega_\infty(1 + 0.8/x)$ . We used this large- $x$  result with  $\Omega_\infty$  from (3) at  $N = 3$ . The theoretical and experimental slopes agree remarkably well. The good agreement in the intercept is somewhat surprising as its theoretical value (= 0.8) is known only at  $N = \infty$ .

ment with recent observations [7]. At larger  $T$ , Ref. [23] predicted a crossover to a QC behavior. Complementing this, the present theory predicts that  $1/T_1$  becomes nearly  $T$  independent for  $x > 1$  or  $T > 0.35J$ . This has in fact already been observed in series expansions [8] and finite cluster calculations [10] for the square lattice antiferromagnet. More importantly, a flattening in  $1/T_1(T)$  has recently been observed in the experiments on  $\text{La}_2\text{CuO}_4$  [7]. We also calculated, from our results above, the limiting large- $T$  value of  $1/T_1$  for the same values of parameters as were used in the low- $T$  fit [23] and found  $1/T_1 \approx 3.3 \times 10^3 \text{ sec}^{-1}$ ; this is in good agreement with the experimental result  $1/T_1 \approx 2.7 \times 10^3 \text{ sec}^{-1}$ . Furthermore, the experimental  $T$  range over which  $1/T_1$  is nearly  $T$  independent increases upon doping. This is consistent with our results because  $\rho_s$  is expected to decrease with doping, thus pushing the system into larger  $x$  for the same  $T$ .

Finally, the correlation length  $\xi$ . Detailed measurements of  $\xi(T)$  in  $\text{La}_2\text{CuO}_4$  have been performed at low  $T$ , where the system is in the RC region [6]. At the highest experimentally accessible  $T$  ( $= 560 \text{ K}$  for  $J = 1460 \text{ K}$ ), our result  $\xi^{-1} = 0.023 \text{ \AA}^{-1}$  is not far from the experimental value of  $\xi^{-1} = 0.03 \pm 0.004 \text{ \AA}^{-1}$ . At finite doping, we expect the crossover between two regimes to occur at lower  $T$ ; QC behavior should therefore be observable at  $T$  even below  $500 \text{ K}$ . We fitted the data of [6] at  $x = 0.04$  by Eq. (6) and found satisfactory agreement with the data over the  $T$  range between  $300$  and  $550 \text{ K}$ . At lower  $T$ , the experimental results on the dynamical local susceptibility [6,11] clearly show that the effects of randomness are relevant. We also compared our results with the numerical data for  $\xi$  at higher  $T$  [9,26]. For  $T > 0.35J$  these data obey quite well  $\xi^{-1} \propto x(1 - \gamma/x)$  where  $\gamma$  is close to 1. However, the overall factor in  $\xi^{-1}$  in the fit is close to twice our  $N = \infty$  result. This discrepancy is probably due to the fact that the strong singular corrections in  $X_Q(x)$  at small  $x$  cause the crossover from small- to large- $x$  behavior to occur at a larger  $x$ . Note, however, that the Monte Carlo calculations in the QC region [27] yield the value of  $X_\infty = 1.25$ , which is close to our result to order  $1/N$  at  $N = 3$  of  $X_\infty = 1.038$ .

To conclude, we have considered in this paper the magnetic properties of two-dimensional quantum antiferromagnets. We focused attention on the  $T$  range where the classical low- $T$  description is no longer valid and the behavior of observables is governed by the renormalization-group flows near the  $T = 0$  quantum fixed point. A comparison with the experimental data for the uniform susceptibility and  $^{63}\text{Cu}$  spin-lattice relaxation rate shows that the intermediate behavior has been observed in the range  $0.35J < T < 0.55J$  in  $\text{La}_2\text{CuO}_4$ .

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[1] See, e.g., S. Chakravarty, in *High-Temperature Supercon-*

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**ERRATA**


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**Localization in a Random Magnetic Field in 2D**  
**[Phys. Rev. Lett. 70, 1980 (1993)]**

Taishi Sugiyama and Naoto Nagaosa

Thirteen lines above Eq. (2) we have erroneously quoted the results by Kalmeyer and Zhang [1] as they concluded that all the states are extended. In Ref. [1] they studied the states at the center of the band ( $E=0$ ) and concluded that they are extended. In a recent paper [2], they have studied the energy dependence and concluded the mobility edge. Their raw data [2] are consistent with ours, but their conclusion is different from ours; i.e., all the states are localized.

[1] V. Kalmeyer and S. C. Zhang, Phys. Rev. B **46**, 9889 (1992).

[2] V. Kalmeyer, D. Wei, D. P. Arovas, and S. C. Zhang (unpublished).

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**Universal Magnetic Properties of  $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$  at Intermediate Temperatures**  
**[Phys. Rev. Lett. 71, 169 (1993)]**

Andrey V. Chubukov and Subir Sachdev

Our conjecture about the structure of the *subleading* terms in the quantum-critical region is incorrect. The large  $x = Nk_B T / 2\pi\rho_s$  behavior of the universal function  $\Omega_Q(x)$  for the uniform susceptibility [ $\chi_u(T) = (g\mu_B/\hbar c)^2 \times k_B T \Omega_Q(x)$ ] can, in general, be written in the form  $\Omega_Q = \Omega_\infty + \Omega_1/x^\alpha + \dots$ . We performed explicit computations of  $\Omega_\infty$  to order  $1/N$ , and of  $\Omega_1, \alpha$  at  $N = \infty$ . We obtained  $\alpha = 1 + \mathcal{O}(1/N)$  and conjectured that there were no  $1/N$  corrections to  $\alpha$ . Chakravarty has pointed out to us, and we have determined by explicit computation, that this conjecture is incorrect. We have instead  $\alpha = 1/\nu = 1 + 32/(3\pi^2 N) + \dots$ . The point is that the function  $\Omega_Q(x)$  is analytic at large  $x$  in the *bare* coupling  $g - g_c$  and not, as we suggested, in the renormalized coupling  $\rho_s \sim (g_c - g)^\nu$  (though  $\Omega_Q$  depends *only* on the fully renormalized  $\rho_s$ ). However, the effects of including this additional  $1/N$  correction in the comparison with data are in fact quite small at  $x \geq 1$ —the change in  $\chi_u$  even at the largest  $T = 0.6J$  is less than 6% compared to our previous expression; errors of similar magnitude are made by neglecting terms higher order in  $1/x$  in the large  $x$  expansion. The use of  $\alpha = 1$  is therefore still justified for experimental comparisons in the quantum-critical region  $0.35J < T < 0.6J$ .

Similar considerations also apply to the function  $X_Q(x)$  for the correlation length.