Holography of the Dirac Fluid in Graphene with two currents

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Recent experiments have uncovered evidence of the strongly coupled nature of the graphene: the Wiedemann-Franz law is violated by up to a factor of 20 near the charge neutral point. We describe this strongly-coupled plasma by a holographic model in which there are two distinct conserved U(1) currents. We find that our analytic results for the transport coefficients for two current model have a significantly improved match to the density dependence of the experimental data than the models with only one current. We also discuss the origin of the two currents.

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Introduction: It has been argued that graphene near charge neutrality forms a strongly interacting plasma, the Dirac fluid. It does not have well-defined quasiparticle excitations, and amenable to a hydrodynamic description [1–10]. Evidence for such a Dirac fluid has appeared in recent experiments [11] on a violation of the Wiedemann-Franz law (WFL) in extremely clean graphene near the charge neutral point: the ratio of heat conductivity and electric conductivity, $\mathcal{L} = \kappa/T\sigma$, was found to be up to 20 times the Fermi liquid value.

The simplest hydrodynamic model [12], with point-like and uncorrelated disorder and a single conserved U(1) current, agrees with the overall experimental trends, but has difficulty capturing the density dependencies of both the electrical (σ) and thermal (κ) conductivities [13]. An alternative hydrodynamic model, the "puddle" model, with long-wavelength disorder in the chemical potential and a single conserved U(1) current, led to a better agreement with observations [13], but still left a room for improvement.

In this letter, we will explore a model with two conserved U(1) currents, motivated by the possibility that there is an extra current that carries mostly heat. Our model will be formulated in holographic terms[14, 15], to utilise the recent progress in the developoment of transport calculation in gauge/gravity duality [16–26]. The Dirac fluid in our model is described by an Anti de Sitter (AdS) black hole in 3+1 dimensions, the holographic dual of 2+1 dimensional system at finite temperature. The momentum dissipation is treated using scalar 'axion' fields, which corresponds to weak point-like disorder. We calculate electric, thermo-electric power and thermal conductivities analytically. We find that, under the assumption that the conserved charges Q_1, Q_2 are proportional to each other, the theoretical results for the density dependencies of the electric and heat conductivities can now satisfactorily match the the experimental data in the Dirac fluid regime.

One possible mechanism for the extra current is the kinematic constraints of energy-momentum conservation on the Dirac cone, which reduce the phase space of electron and hole scattering significantly [4], allowing electrons and holes to form independent currents as far as the relaxation time for mixing between the currents is presumed to be is much longer than the Plankian relaxation time $\hbar/k_BT$, the time required for hydrodynamic regime at work. It should be noted, however, that the estimates of electron and hole equilibration times are made in quasiparticle framework [4], whose validity in hydrodynamic regime is just assumed here. We will see that the kinematics on the Dirac cone also provide a reason why the two charge densities can be proportional.

DC Transport with two $U(1)$ fields : We start from the action $S = \int d^4x\sqrt{-g}\mathcal{L}$ with two gauge fields A_μ, B_μ, a dilaton field ϕ and the scalar fields χ_1, χ_2 for momentum dissipation:

$$\begin{aligned}\mathcal{L} =& R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi_1(\phi)(\partial\chi_1)^2 + \Phi_2(\phi)(\partial\chi_2)^2\right] \\ &- V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{W(\phi)}{4}G^2, \end{aligned} \tag{1}$$

where $F = \mathrm{d}A, \quad G = \mathrm{d}B$ and $F^2 = F_{\mu\nu}F^{\mu\nu}$ etc. We also require positivity of $\Phi_i(\phi)$, $Z(\phi)$ and $W(\phi)$. We take the ansatz for background metric and the gauge potential as

$$\begin{aligned} ds^2 &= -U(r)dt^2 + \frac{1}{U(r)}dr^2 + e^{v(r)}(dx^2 + dy^2) \\ A &= A(r)dt, \quad B = B(r)dt. \end{aligned} \tag{2}$$

The gauge field A has the chemical potential and charge density as its components of its near boundary expansion, $A(r) = \mu_1 - Q_1/r + \cdots$. Near the horizon at $r = r_0$,

$$\begin{aligned} &U \sim 4\pi T(r - r_0) + \cdots, \ A(r) \sim A_+(r - r_0) + \cdots, \\ &B(r) \sim B_+(r - r_0) + \cdots, \end{aligned} \tag{3}$$

for the regularity. If we take the axion solution, $\chi_1 = kx, \ \chi_2 = ky$, it provides momentum relaxation. From now on, we set $\Phi_1 = \Phi_2 = \Phi$ for simplicity. The only non-zero components in the Maxwell equations are that for the tr-component of the field strengths whose first integral give conserved charges,

$$\begin{aligned} Q_1 &= \sqrt{-g}Z(\phi)F^{tr} = Z(\phi)e^v A'(r) \\ Q_2 &= \sqrt{-g}W(\phi)G^{tr} = W(\phi)e^v B'(r). \end{aligned} \tag{4}$$

One can see that if $e^v \sim r^2$ in asymptotic region, Q_i corresponds to the number density of the boundary field theory. To compute the transport coefficients, we turn on small fluctuations around the background solution:

$$\begin{aligned}\delta G_{tx} =& t\delta f_1(r) + \delta g_{tx}(r), \quad \delta G_{rx} = e^{v(r)}\delta h_{rx}(r),\\ \delta A_x =& t\delta f_2(r) + \delta a(r), \quad \delta B_x = t\delta f_3(r) + \delta b(r),\end{aligned} \quad (5)$$

as well as $\delta\chi_i(r)$'s. We choose the functions $f_i(r)$ as

$$\begin{aligned}&\delta f_1(r) = -\zeta U(r), \quad \delta f_2(r) = -E_1 + \zeta A(r),\\ &\delta f_3(r) = -E_2 + \zeta B(r),\end{aligned} \quad (6)$$

such that the time t does not appear in the equations of motion of the fluctuations. Here, E_1, E_2 are thmo-electric forces acting on J_1, J_2 respectively and $\zeta = -\nabla T/T$. We can define currents as [18],

$$\begin{aligned}&J_1 = \sqrt{-g}Z(\phi)F^{xr}, \qquad J_2 = \sqrt{-g}W(\phi)G^{xr}\\ &\mathcal{Q} = U(r)^2\frac{d}{dr}\left(\frac{\delta g_{tx}(r)}{U(r)}\right) - A(r)J_1 - B(r)J_2.\end{aligned} \quad (7)$$

Notice that near the boundary, the heat current becomes $\mathcal{Q} = T^{tx} - \mu_1 J_1 - \mu_2 J_2$. Moreover, these currents are conserved along radial direction r. Therefore their boundary values are related to that of horizon data:

$$J_1(\infty) = J_1(r_0), \; J_2(\infty) = J_2(r_0), \; \mathcal{Q}(\infty) = \mathcal{Q}(r_0). \quad (8)$$

Then, the boundary value of currents are

$$\begin{aligned}J_1(\infty) &= -e^{v_0}Q_1\delta g_{tx}^{(0)} + E_1 Z_0\\ J_2(\infty) &= -e^{v_0}Q_2\delta g_{tx}^{(0)} + E_2 W_0\\ \mathcal{Q}(\infty) &= -4\pi T\delta g_{tx}^{(0)},\end{aligned} \quad (9)$$

where subscript 0 denotes the value at the horizon. $\delta g_{tx}^{(0)}$ can be obtained by the horizon data from fluctuation equation together with the regularity condition,

$$\delta g_{tx}^{(0)} = -(E_1Q_1 + E_2Q_2 + 4\pi T e^{v_0}\zeta)/(k^2\Phi_0), \quad (10)$$

with $\zeta = -\frac{\nabla T}{T}$. Finally, we get the boundary current in terms of the external sources:

$$\begin{aligned}\mathcal{Q} &= \frac{4\pi T Q_1}{k^2\Phi_0}E_1 + \frac{4\pi T Q_2}{k^2\Phi_0}E_2 + \frac{(4\pi T)^2 e^{v_0}}{k^2\Phi_0}\zeta,\\ J_1 &= \left(Z_0 + \frac{e^{-v_0}Q_1^2}{k^2\Phi_0}\right)E_1 + \frac{e^{-v_0}Q_1Q_2}{k^2\Phi}E_2 + \frac{4\pi T Q_1}{k^2\Phi_0}\zeta,\\ J_2 &= \left(W_0 + \frac{e^{-v_0}Q_2^2}{k^2\Phi_0}\right)E_2 + \frac{e^{-v_0}Q_1Q_2}{k^2\Phi}E_1 + \frac{4\pi T Q_2}{k^2\Phi_0}\zeta.\end{aligned} \quad (11)$$

The eq. (11) can be written in matrix form,

$$\mathcal{J} = \Sigma\cdot\mathcal{S}, \quad (12)$$

where $\mathcal{J} = (\langle\mathcal{Q}\rangle,\ \langle J_1\rangle,\ \langle J_2\rangle)^T$ and $\mathcal{S} = (\zeta,\ E_1,\ E_2)^T$ are current and the external source multiplets respectively. The transport coefficients are defined by

$$\begin{pmatrix}\bar\kappa T & \bar\alpha_1 T & \bar\alpha_2 T\\ \alpha_1 T & \sigma_1 & \bar\delta\\ \alpha_2 T & \delta & \sigma_2\end{pmatrix} := \Sigma. \quad (13)$$

σ_i and α_i are the (partial) electric conductivities and the Seeback coefficients for the current J_i. Since the components of Σ can be read off from (11), we now know all the transport coefficients: $\sigma_1 = Z_0 + \frac{Q_1^2}{k^2e^{v_0}\Phi_0}, \alpha_1 = \frac{4\pi TQ_1}{k^2\Phi_0}$, etc. Notice that the matrix is real and symmetric, so that the Onsager relations hold:

$$\bar\alpha_i = \alpha_i, \quad \bar\delta = \delta. \quad (14)$$

κ is defined by the response of the temperature gradient to the heat current in the absence of other currents: setting J_1 and J_2 to be zero in (11), we can express E_1 and E_2 in terms of ζ. Substituting these to the first line of (11), we get

$$\kappa = \bar\kappa - \frac{T\bar\alpha_1(\alpha_1\sigma_2 - \alpha_2\bar\delta)}{\sigma_1\sigma_2 - \delta\bar\delta} - \frac{T\bar\alpha_2(\alpha_2\sigma_1 - \alpha_1\delta)}{\sigma_1\sigma_2 - \delta\bar\delta}. \quad (15)$$

To discuss more explicitly, we consider a black hole solution with two charges:

$$U(r) = r^2 - \frac{m_0}{r} - \frac{k^2}{2} + \frac{1}{4r^2}\left(Z_0Q_1^2 + W_0Q_2^2\right), \quad (16)$$

where m_0 is given by $U(r_0) = 0$ and the temperature is

$$T = \frac{1}{4\pi}\left(3r_0 - \frac{k^2}{2r_0} - \frac{Z_0Q_1^2}{4r_0^3} - \frac{W_0Q_2^2}{4r_0^3}\right). \quad (17)$$

The solutions of $U(1)$ gauge fields are $a(r) = \mu_1 - \frac{Q_1}{r}$, $b(r) = \mu_2 - \frac{Q_2}{r}$. For the finite vector norm $g^{\mu\nu}A_\mu A_\nu$ at the horizon $r = r_0$, we need $\mu_i = Q_i/r_0$.

The conductivities for any number of conserved currents can be calculated explicitly:

$$\sigma_i = Z_i + \frac{Q_i^2}{r_0^2k^2}, \;\; \sigma_{ij} = \frac{Q_iQ_j}{r_0^2k^2}, \;\; \kappa = \frac{s^2T}{r_0^2k^2 + \sum_i Q_i^2/Z_i},$$

with $s = 4\pi r_0^2$ and Z_i control the coupling of the A_i. For two currents case, Z_1, Z_2 denote Z_0, W_0 respectively. If we identify the total electric current as $J = \sum_i J_i$ and thermo-electric force $E_i = E - T\nabla(\mu_i/T)$, we can calculate the electric conductivity as

$$\sigma = \frac{\partial J}{\partial E} = \sum_i \sigma_i + \sum_{i,j}\sigma_{ij} = Z + Q^2/r_0^2k^2, \quad (18)$$

where $Z = \sum_i Z_i$ and $Q = \sum_i Q_i$. Above formula check the consistency of our transport formula under the virtual division of a system into many subsystem, like electrons belonging to sub-lattices A and B.

Finally we compare our result with the experimental data. We consider two current case with linear relation between the two charges,

$$Q_2 = gQ_1, \tag{19}$$

whose justification will be discussed later. If we choose $Z_0 = W_0 = 1/3$, $k = 3$, $T = 0.17$ and $g = 3$, we can fit the experimental data for charge density dependence of the conductivities near the charge neutral point of graphene without leaving much room to improve as one can see in Figure 1.

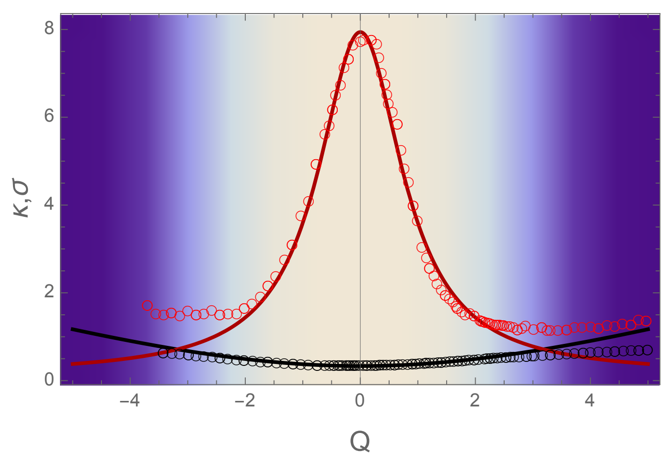

FIG. 1. holography v.s the experimental data: $k = 3, g = 3, T = 0.17$ and $W_0/e^2 = Z_0/e^2 = 1/3$. We rescale the x axes by 100 to fit the experimental data.

Origin of two Currents in Graphene: What is the nature and the origin of the extra current in the graphene. There are a few attractive candidates. The first idea is the effect of imbalance [4] between the electrons and holes due to the kinematical constraints of the Dirac cone. When there is such an deviation of electron and hole density from their equilibrium value, then the system has tendency to reduce the difference by creating/absorbing electron-hole pair:

$$e^- \leftrightarrow e^- + h^+ + e^-, \quad h^+ \leftrightarrow h^+ + h^+ + e^- \tag{20}$$

In such processes, energy and momemtum conservations must hold. The point is that, for the graphene, the linear dispersion relation severely reduces the kinematically available states [4]: If we define $\vec{q}$ as a momentum measured from a Dirac point,

$$\vec{q_1} = \vec{q_2} + \vec{q_3} + \vec{q_4}, \quad |\vec{q_1}| = |\vec{q_2}| + |\vec{q_2}| + |\vec{q_3}|, \tag{21}$$

which request the co-linearity of all momentum vectors $\vec{q}_1, \cdots, \vec{q}_4$. Therefore available phase space is greatly reduced. Such kinematical constraints maintains the non-equilibrium states and as a consequence, the two currents J_e, J_h behave independently for a long time compared with the Planck time $\sim \hbar/kT$, which is the time for hydrodynamics to work.

The net electric current J and total number current J_n which become neutral at Dirac point, are defined by $J = J_e + J_h, \quad J_n = J_e - J_h$, respectively and their electric charge densities and number densities are related by $Q_1 = en_1$ and $Q_2 = -en_2$. The total electric conductivity $\sigma = \frac{\partial J}{\partial E}$ and κ can be expressed in terms of $Q = Q_1 + Q_2$ and $Q_n := Q_1 - Q_2$:

$$\sigma = W_0 + Z_0 + \frac{Q^2}{k^2 r_0^2}, \; \kappa = \frac{(4\pi r_0^2)^2\, T}{r_0^2 k^2 + (Q^2 + Q_n^2)/2Z_0}. \tag{22}$$

where we used charge conjugation symmetry, $W_0 = Z_0$.

Now it is time to ask why we can set the *proportionality of the two charges* as given in eq. (19). First, notice that the presence of inhomogeneity can make this issue subtle, because in that case we can create spatially separated electron-rich and hole-rich regions without breaking charge conservation. Such inhomogeneity is called charge puddle. To avoid the issues involved in the transport by puddle, we simply assume that well localized puddles do not contribute transport or simply assume that the system is homogeneous. Under such assumption, the number densities of electrons and holes created by thermal excitation is proportional to the net charge density: for the fermi liquid case, out of total degree of freedom (d.o.f) $n \sim k_F^2 \sim \mu^2$, excitable d.o.f is $\sim kT \cdot \mu$, because the excitable shell width is kT. But in hydrodynamic regime, $kT >> \mu$, therefore entire non-degenerate charge distribution (NDCD) region is excitable. In fact this is a typical situation of fermion dynamics described by AdS black hole [27, 28]. In summary, in case of the hydrodynamic regime, the charge carrier density created is proportional to total degree of freedom, Q, which is the volume of the Dirac cone above the Dirac point.

We remark that due to strong Coulomb interaction, the created electron hole pairs can form the bound state, exciton. Such excitons in homogeneous graphene satisfies the linear relations between the electric charge and the exciton number. Although exciton in graphene has been discussed extensively [29, 30], mosts are only for bi-layer graphene. However, we expect that strong coulomb interaction in Dirac Fluid regime of single layer graphene should be able to make bound state. The abundance of such excitons are remained to be verified experimentally.

Discussions: In the presence of an extra current that carries mainly heat, the violation of WFL is not direct evidence of a Dirac fluid. However, the fact that such a phenomenon is quantitatively well described by hydrodynamics and gauge/gravity duality, indicates that the system is strongly correlated.

Disorder and the nature of axion: The axion provides momentum dissipation only when both its gradient and the vacuum expectation value of its dual operator, $\langle \mathcal{O}_I \rangle$, are nonzero. The latter is the analogue of charge density in electric field as one can see from the Ward identity,

$$\nabla_\nu T^{\mu\nu} = \langle \mathcal{O}_I \rangle \nabla_\mu \chi_I^0 + F^0_{\mu\nu} \langle J^\nu \rangle . \tag{23}$$

The role of the source field $\chi_I^0 = kx_I$ is the chemical potential of impurity and that of $\langle \mathcal{O}_I \rangle$ is the density of impurity. Therefore, k^2 can be understood as the density of the uniform impurity.

The puddle effect on the transport: One important source of the disorder in graphene is known to be the the charge density inhomogeneity, [11, 13, 31], which is completely neglected here. Then why the theory could match the experiment so well? A partial answer is that while the DC conductivities directly depends only on the average density of the impurity, as one can see from the Kubo formula of the conductivity[32]:

$$\sigma_{ij}(\omega) = \lim_{q\to 0} \frac{1}{i\omega} \langle J_i(q,\omega) J_j(-q,-\omega) \rangle , \tag{24}$$

where $J_i(q)$ is the Fourier transform of the the current operator $J_i(x)$. In leading order, the conductivity depends only on the $\langle J_0 \rangle$, the spatial average of the $\langle J(x) \rangle$. Although both the imbalance and the inhomogeneity effects contribute to the details of transport coefficients, if we use best fitting method, considering both effects can generate double counting. Therefore it is quite possible that there is no room for puddle effect after we consider the more direct effect.

Other origins of the second current: We suggested imbalacnce and excitons as possible sources for the extra current. Here we discuss other candidates. *i) Spin charge separation*:This is the simplest to explain the phenomena if such separation could be experimentally confirmed: the spinons are obviously the chargeless heat carrier and densities of spinnons and holons must be the same and equal to the original electron density. *ii) valley currents*: Graphene consists of two sublattices A and B and such electrons in each sublattice do not scatter, hence they form two conserved currents. However, they do not necessarily satisfy the linearity condition eq.(19). *iii) Phonon*: At high temperature, the phonons are the main heat carriers in carbon materials. However, there are good reasons that phonon is not the main player in the regime we are discussing [11].

Future directions: It would be interesting if we can extend our method to multilayered graphene and graphite. Some holographic analysis for the latter was already reported [26]. The thermo-electric power and magneto-transport are also very interesting observable for the Dirac Fluid regime. We note that some of the early data began to be produced[33]. From the experimental side, the abundance of excitons in single layer graphene is remained to be verified experimentally.

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Appendix A: Mathematical Supplements

Here we collect some of the equations which are helpful to check the formula in this paper. The action (1) yields equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{L} - T_{\mu\nu} = 0, \qquad \frac{1}{\sqrt{-g}}(\Phi_i(\phi)\nabla^\mu\chi_i) = 0,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}Z(\phi)F^{\mu\nu}) = 0, \ \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}W(\phi)G^{\mu\nu}) = 0,$$

$$\nabla^2\phi - \frac{1}{2}\sum_{i=1}^{2}\frac{\partial\Phi_i(\phi)}{\partial\phi}(\partial\chi_i)^2 - \frac{\partial V(\phi)}{\partial\phi} - \frac{1}{4}F^2\frac{\partial Z(\phi)}{\partial\phi} - \frac{1}{4}G^2\frac{\partial W(\phi)}{\partial\phi} = 0, \quad \text{(A1)}$$

$$T_{\mu\nu} = \frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi) + \sum_{i=1}^{2}\frac{1}{2}\Phi_i(\phi)(\partial_\mu\chi_i)(\partial_\nu\chi_i) + \frac{Z(\phi)}{2}F_\mu^\lambda F_{\nu\lambda} + \frac{W(\phi)}{2}G_\mu^\lambda G_{\nu\lambda}. \quad \text{(A2)}$$

From the equations for metric and the dilaton, we can solve for v'', U'', U' to give

$$\begin{aligned}
v''(r) &= -\frac{1}{2}(v'(r)^2 + \phi'(r)),\\
U'(r) &= -\frac{e^{-2v}}{2ZWv'(r)}\Big[Q_1^2W + Q_2^2Z + 2e^vVZW\\
&\quad + e^vZW(2k^2\Phi + e^vU(r)(v'(r) - \phi'(r))\Big],\\
U''(r) &= e^{-2v}\left(\frac{Q_1^2}{Z} + \frac{Q_2^2}{W}\right) + e^{-v}k^2\Phi + \frac{1}{2}U(r)(v'(r)^2 - \phi'(r)).
\end{aligned}$$

With the choices (6), the Einstein equations for the fluctuation becomes

$$\begin{aligned}
&2U(r)\delta g_{tx}''(r) = \big(2e^{-v}k^2\Phi(\phi) + U(r)(v'(r)^2 - \phi'(r)^2)\big)\delta g_{tx}(r)\\
&- 2e^{-v}U(r)(Q_1\,\delta a'(r) + Q_2\,\delta b'(r))\\
&\delta h_{rx}(r) = -\frac{e^{-v}}{k^2U(r)\Phi(\phi)}\Big[EQ_1 + E_2Q_2 - e^vkU(r)\Phi\delta\chi'(r)\\
&+ (e^vU'(r) - e^vU(r)v'(r) - Q_1A(r) - Q_2B(r))\zeta\Big],
\end{aligned} \quad \text{(A3)}$$

where we used the background solution In the presence of the black hole horizon, every equations of motion should be regular at the black hole horizon. The fluctuations should behave[19]

$$\begin{aligned}
\delta a(r) &\sim -\frac{E_1}{4\pi T}\ln(r - r_0) + \cdots,\\
\delta b(r) &\sim -\frac{E_2}{4\pi T}\ln(r - r_0) + \cdots,\\
\delta g_{tx}(r) &\sim \delta g_{tx}^{(0)} + \mathcal{O}((r - r_0)) + \cdots,\\
\delta h_{rx}(r) &\sim e^{-v(r_0)}\frac{\delta g_{tx}^{(0)}}{U(r_0)} + \cdots,\\
\delta\chi_i(r) &\sim \chi_i^{(0)} + \mathcal{O}((r - r_0)) + \cdots.
\end{aligned} \quad \text{(A4)}$$