Signatures of the nematic ordering transitions in the thermal conductivity of d-wave superconductors

Lars Fritz and Subir Sachdev
Department of Physics, Harvard University, Cambridge MA 02138, USA
(Dated: January 22, 2009)

We study experimental signatures of the Ising nematic quantum phase transition in d-wave superconductors, associated with the change of lattice symmetry from tetragonal to orthorhombic in the superconducting state. The characteristic feature of this transition is that the ratio between the Fermi velocity v_F and gap velocity v_Δ flows to a maximally anisotropic fixed point, *i.e.* the renormalization group fixed point is situated at $(v_\Delta/v_F)^* = 0$. Our main point is that the logarithmic approach to this fixed point has visible signatures in the thermal transport. The analysis of the quasiparticle contribution to the thermal transport is carried out in the framework of a kinetic approach, which shows that the thermal conductivity is enhanced near the nematic critical point. Another aspect of our study is the interplay of dilute disorder and electronic interactions in the measured thermal transport coefficients.

PACS numbers:

I. INTRODUCTION

Electronic nematic phases were discussed in the context of doped Mott insulators[1], and have by now been experimentally observed in a number of systems. They have been observed in semiconductor heterostructures[2], in the bulk transition metal oxide $Sr_3Ru_2O_7$[3], as well as in YBCO. In all these cases the thermal transition to the nematic phase seems to be second order, whereas the nature of the quantum phase transition is much less clear. In the case of $Sr_3Ru_2O_7$ it seems to be first order[3].

Now there is evidence that a nodal nematic phase occurs in at least some of the underdoped cuprate superconductors. In the nematic phase, the square lattice symmetry is broken down to tetragonal symmetry, a consequence of the instability of the interacting electronic system to partial stripe-like order. The best evidence for this comes from measurements of strongly temperature dependent transport anisotropies in underdoped $YBa_2Cu_3O_{6+\delta}$[4] and from neutron scattering experiments in underdoped $YBa_2Cu_3O_{6.45}$[5].

On the theoretical side, Vojta *et al.*[6] analyzed possible quantum phase transitions in d-wave superconductors in the framework of a renormalization group (RG) analysis. An initial RG analysis found runaway flow for the nematic ordering instability at zero temperature based on an expansion in $3-d$, where d denotes the spatial dimensionality. In a recent work, Kim *et al.*[7] found in the framework of a large-N analysis the existence of a second order transition to a nematic phase. Based on an RG analysis in the large-N_f framework, where N_f denotes the number of electronic spin components (the physical case corresponding to $N_f = 2$), Huh[8] *et al.* confirmed the existence of such a second order transition. The authors found an RG fixed point at order $1/N_f$ describing a second-order quantum phase transition associated with the onset of long-range nematic order. The scaling properties near the fixed point are very peculiar. There is a dangerously irrelevant parameter v_Δ/v_F, where v_Δ and v_F are the velocities of the nodal fermions parallel and perpendicular to the Fermi surface, which controls the fixed point. The fixed point lies at "infinite anisotropy", *i.e.* $(v_\Delta/v_F)^* = 0$, which has to be contrasted from the other relativistic fixed points found for other competing orders[6]. In order to calculate physical quantities one has to use a fully two-dimensional theory, since the flow of the anisotropy to zero is logarithmically slow as a function of the relevant energy scale, as is described by Huh *et al.*.[8]

In the present paper we study thermal transport properties at the nematic to isotropic quantum phase transition (QPT) deep within the d-wave superconducting phase of a quasi two dimensional tetragonal crystal[7] in the framework of the Boltzmann equation. The main result of our analysis is the logarithmic enhancement of the thermal conductivity upon lowering the temperature.

The paper is organized as follows. We start with a general review of the model for the second order nematic phase transition and its properties in a few limiting cases in Sec. II. This closely follows the presentation in Refs. 7,8. We proceed with the definition of the heat current operator of the electronic quasiparticles in Sec. III. This discussion is complemented by the discussion on the bosonic contribution to the thermal conductivity presented in Appendix D. In Sec. IV we introduce the Boltzmann equation framework and explain the formalism used to solve. This discussion uses a lot of symmetry properties, which are more explicitly detailed in Appendix A. In Sec. IV A we review a Boltzmann equation analysis of a disordered d-wave superconductor *without* a nematic mode, which makes connection to existing results[12,19]. Then we analyze the full problem in Sec. IV B. The numerical solution of the Boltzmann equation is presented in Sec. V B and several different situations are discussed, such as the interplay of inelastic and elastic scattering. Finally, in Sec. VI we conclude and comment on possible experimental implications of our analysis. Appendix C provides some additional infor-

mation on transport in clean systems and complements the discussion in Sec. III and Appendix D.

II. THE MODEL

The model under consideration throughout this work has been discussed in the literature in great detail[7,8]. Therefore we will only repeat the key features. The relevant low-energy description of the electronic system in a two-dimensional d-wave superconductor with a pure $d_{x^2-y^2}$ pairing symmetry is given by the following BCS-type Hamiltonian

$$\begin{aligned} H &= \sum_{\mathbf{k},\sigma} \Psi^\dagger_{1\sigma\mathbf{k}} \begin{pmatrix} \mathbf{v}_F^1\cdot\mathbf{k} & \mathbf{v}_\Delta^1\cdot\mathbf{k} \\ \mathbf{v}_\Delta^1\cdot\mathbf{k} & -\mathbf{v}_F^1\cdot\mathbf{k}\end{pmatrix} \Psi_{1\sigma\mathbf{k}} \\ &+ \sum_{\mathbf{k},\sigma} \Psi^\dagger_{2\sigma\mathbf{k}} \begin{pmatrix} \mathbf{v}_F^2\cdot\mathbf{k} & \mathbf{v}_\Delta^2\cdot\mathbf{k} \\ \mathbf{v}_\Delta^2\cdot\mathbf{k} & -\mathbf{v}_F^2\cdot\mathbf{k}\end{pmatrix} \Psi_{2\sigma\mathbf{k}} \,, \end{aligned} \tag{2.1}$$

where the dispersion has been linearized around the four nodal points. The Fermi velocities $\mathbf{v}_F^1$, $\mathbf{v}_F^2$ and the gap velocities $\mathbf{v}_\Delta^2$, $\mathbf{v}_\Delta^2$ are defined as

$$\mathbf{v}_F^1 = \frac{\partial\epsilon_\mathbf{k}}{\partial\mathbf{k}}|_{\mathbf{k}=\mathbf{K}_1} \quad \mathbf{v}_\Delta^1 = \frac{\partial\Delta_\mathbf{k}}{\partial\mathbf{k}}|_{\mathbf{k}=\mathbf{K}_1} \tag{2.2}$$

and

$$\mathbf{v}_F^2 = -\overline{v}\mathbf{v}_\Delta^1 \quad \mathbf{v}_\Delta^2 = \overline{v}^{-1}\mathbf{v}_F^1 \quad \text{with} \quad \overline{v} = \frac{v_F}{v_\Delta} \,, \tag{2.3}$$

where the vectors $\mathbf{K}_i$ denote the location of the nodal points in the Brillouin zone in clock-wise direction starting with $\mathbf{K}_1$ lying at $(\pi/2,\pi/2)$. Furthermore, for later convenience, we introduced the anisotropy parameter $\overline{v} = v_F/v_\Delta$ in the above equations. This parameter plays a vital role since it is a direct measure of the velocity anisotropy and has a nontrivial flow under the renormalization group transformation[8]. The results relevant for the calculations performed in this paper are repeated in Appendix B. Furthermore, it is notationally very convenient to introduce the nodal fermions, called f_1, f_2, f_3, and f_4 living at the respective nodes in k-space. The Nambu spinors are composed of the nodal fermions in the following form

$$\Psi_{1\sigma\mathbf{k}} = \begin{pmatrix} f_{1\sigma\mathbf{k}} \\ \epsilon_{\sigma,-\sigma} f^\dagger_{3-\sigma-\mathbf{k}}\end{pmatrix} \quad \Psi_{2\sigma\mathbf{k}} = \begin{pmatrix} f_{2\sigma\mathbf{k}} \\ \epsilon_{\sigma,-\sigma} f^\dagger_{4-\sigma-\mathbf{k}}\end{pmatrix} \,, \tag{2.4}$$

where $\epsilon_{\sigma,-\sigma}$ is the antisymmetric tensor and σ is a generalized flavor index, which will finally allow the large-N treatment in the limit of a large number of flavors, denoted N_f.

The electronic Hamiltonian in Eq. (2.1) has a Dirac Hamiltonian structure and is readily diagonalized with a standard Bogoliubov transformation. This is achieved by a transformation according to

$$\begin{aligned} \gamma^\dagger_{1\sigma\mathbf{k}} &= u_\mathbf{k} f^\dagger_{1\sigma\mathbf{k}} - v_\mathbf{k} f_{3-\sigma-\mathbf{k}} \,, \\ \gamma_{3-\sigma-\mathbf{k}} &= u_\mathbf{k} f_{3-\sigma-\mathbf{k}} + v_\mathbf{k} f^\dagger_{1\sigma\mathbf{k}} \,, \end{aligned} \tag{2.5}$$

and respectively for nodes 2 and 4; $u_\mathbf{k}$ and $v_\mathbf{k}$ are the coherence factors[9]. The corresponding unitary matrix is given by

$$U^{-1}_{a\mathbf{k}} = \frac{1}{2\epsilon^a(\mathbf{k})} \begin{pmatrix} \mathbf{v}^a_\Delta\mathbf{k} & -\mathbf{v}^a_\Delta\mathbf{k} \\ \epsilon_a(\mathbf{k}) - \mathbf{v}^a_F\mathbf{k} & \epsilon_a(\mathbf{k}) + \mathbf{v}^a_F\mathbf{k}\end{pmatrix} \,. \tag{2.6}$$

where the energies for the Bogoliubov quasiparticles are given by

$$\epsilon_i(\mathbf{p}) = \begin{cases} \sqrt{\left(\mathbf{v}_F^1\cdot\mathbf{p}\right)^2 + \left(\mathbf{v}_\Delta^1\cdot\mathbf{p}\right)^2} & i = 1,3 \\ \sqrt{\overline{v}^2\left(\mathbf{v}_\Delta^1\cdot\mathbf{p}\right)^2 + \overline{v}^{-2}\left(\mathbf{v}_F^1\cdot\mathbf{p}\right)^2} & i = 2,4 \end{cases} \tag{2.7}$$

In our later discussion of the thermal transport properties in terms of the Boltzmann equation it is mandatory to work in the basis of these quasiparticles[23,24].

In the theory considered[6,7,8] an Ising-symmetric nematic order parameter couples to the nodal fermions. The corresponding interaction term has the form of an additional s-wave order parameter, whose condensation has the effect of breaking the four-fold rotation symmetry in k-space down to a two-fold one[6,7]. Consequently, it assumes the following form

$$\mathcal{S}_{\text{int}} = \lambda \sum_{\sigma=1}^{N_f} \int d^2x d\tau \phi \left(\Psi^\dagger_{1\sigma}\tau^x\Psi_{1\sigma} + \Psi^\dagger_{2\sigma}\tau^x\Psi_{2\sigma}\right) \tag{2.8}$$

Kim *et al.*[7] showed that the leading quantum fluctuations at the large-N level after a quadratic expansion around the saddle point lead to a non-analytic form of the effective bosonic theory. This calculation is a straightforward but tedious calculation[10,11], relying on the appropriate usage of Feynman parameters. Retaining only the terms which are relevant at low energies in the RG sense (assuming the mass of the bosonic action is tuned to criticality), the effective bosonic action assumes the following form

$$\begin{aligned} \mathcal{S}_\phi &= \frac{\gamma}{2\beta}\sum_n \int \frac{d^2k}{(2\pi)^2} \left(\frac{\omega_n^2 + \epsilon_1^2(\mathbf{p}) - \left(\mathbf{v}^1_\Delta\mathbf{p}\right)^2}{\sqrt{\omega_n^2+\epsilon_1^2(\mathbf{p})}} \right. \\ &+ \left. \frac{\omega_n^2 + \epsilon_2^2(\mathbf{p}) - \overline{v}^{-2}\left(\mathbf{v}^1_F\cdot\mathbf{p}\right)^2}{\sqrt{\omega_n^2+\epsilon_2^2(\mathbf{p})}} \right) |\phi(\mathbf{p})|^2 \,, \end{aligned} \tag{2.9}$$

where

$$\gamma = \frac{\lambda^2}{32 v_F v_\Delta} \,. \tag{2.10}$$

This implies that the effective propagator of the bosonic modes is given by

$$\begin{aligned} D^{\omega_n}_\mathbf{k} &= \frac{1}{\gamma} \left(\frac{\omega_n^2 + \epsilon_1^2(\mathbf{p}) - \left(\mathbf{v}^1_F\mathbf{p}\right)^2}{\sqrt{\omega_n^2+\epsilon_1^2(\mathbf{p})}} \right. \\ &+ \left. \frac{\omega_n^2 + \epsilon_2^2(\mathbf{p}) - \overline{v}^2\left(\mathbf{v}^1_\Delta\cdot\mathbf{p}\right)^2}{\sqrt{\omega_n^2+\epsilon_2^2(\mathbf{p})}} \right)^{-1} \,. \end{aligned} \tag{2.11}$$

Higher order terms are as usual in large-N theories down in powers of $1/N_f$. At this point it is instructive to discuss the above propagator in two extreme cases, namely for $\overline{v}=1$, *i.e.* isotropic velocities, and for $\overline{v}\gg 1$.

For $\overline{v}=1$ the corresponding bosonic propagator is simply given by

$$D_{\mathbf{k}}^{\omega_n} \;=\; \frac{1}{\gamma}\left(\frac{2\omega_n^2+\epsilon^2(\mathbf{p})}{\sqrt{\omega_n^2+\epsilon^2(\mathbf{p})}}\right)^{-1} \;, \qquad (2.12)$$

where the energies ϵ_1 and ϵ_2 are now identical due to the perfect isotropy. It is important to note that the present propagator implies the existence of quasiparticle-like peaks sitting on top of a continuum in the spectral function[7]. In the limit of extreme anisotropy ($\overline{v}\gg 1$), which is the main focus in this paper since the RG fixed point sits at infinite anisotropy, the propagator reads

$$D_{\mathbf{k}}^{\omega_n} = \frac{1}{\gamma}\left(\sqrt{\omega_n^2+(\mathbf{v}_F^1\cdot\mathbf{p})^2}+\sqrt{\omega_n^2+(\mathbf{v}_F^2\cdot\mathbf{p})^2}\right)^{-1} \qquad (2.13)$$

This propagators plays an important role in determining the contribution of the bosonic sector to thermal transport, which we discuss in Appendix D. It was already pointed out that the presence of the nematic order parameter and its interaction with the Bogoliubov quasiparticles leads to a finite quasiparticle lifetime[7].

In order to discuss transport in a realistic setting we will also add some dilute disorder. On a Hamiltonian level this reads

$$H_{\rm dis} = \int \sum_{i,j,\sigma} \frac{d^2k}{(2\pi)^2}\frac{d^2k'}{(2\pi)^2} V^{ij}_{\mathbf{k}\mathbf{k}'} f^\dagger_{i\sigma\mathbf{k}} f_{j\sigma\mathbf{k}'} \;, \qquad (2.14)$$

where $V^{ij}_{\mathbf{k}\mathbf{k}'}$ stands for the scattering matrix element of an electron living around node i with momentum $\mathbf{k}$ into an electron living around node j with momentum $\mathbf{k}'$. In contrast to Refs. 12,13 we assume a source of isotropic scattering, which implies that $V^{ij}_{\mathbf{k}\mathbf{k}'}=V_{\mathbf{k}\mathbf{k}'}=\hat{u}$. However, all the results of the following calculations can in principle be extended to account for the more generic case including different scattering strenghts.

III. DERIVATION OF THE HEAT CURRENT OPERATOR

The derivation of the heat current operator in interacting electronic systems is a long-standing problem stemming from the fact that a temperature gradient cannot be represented as a mechanical perturbation[29,31,35,41]. Commonly, a Lagrangian approach is adopted to derive the appropriate relations. In the case of an electronic system interacting with a bosonic system, the microscopic expression for the heat current was derived for the case of phonons by Vilenkin *et al.* [28]. This procedure, however, does not exactly apply to our problem, due to the fact that the effective low-energy action of the bosonic mode is created by the electrons themselves. In general, for the system described by Eqs. (2.1), (2.8), and (2.9) it is practically impossible to disentangle the contributions to the energy current due to the electronic and the bosonic degrees of freedom. We will further address this issue in Appendix D.

First, we will start with the contribution carried solely by the quasiparticles, which will constitute the basis for a Boltzmann equation approach. For a general superconductor, irrespective of the pairing symmetry, we can refer the reader to Refs. 12,19,20,25,31 for a proper derivation of the appropriate operator. The final expression for the heat current carried by the Bogoliubov quasiparticles is readily given by the intuitive expression

$$\mathbf{j}_E = \sum_{\sigma=1}^{N_f}\sum_{i=1}^{4}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\epsilon_i(\mathbf{k})\frac{\partial\epsilon_i(\mathbf{k})}{\partial\mathbf{k}} f^i_\sigma(\mathbf{k}) \;. \qquad (3.1)$$

This expression will constitute the starting point for our Boltzmann transport equation analysis in Sec. IV.

IV. BOLTZMANN TRANSPORT EQUATIONS

Within this section we access the thermal transport properties using the semiclassical Boltzmann-equation approach. This approach has proven to be a powerful tool to compute transport properties of quantum critical systems[14,15,21,22,23,24].

The central object in Boltzmann transport theory is the distribution matrix of the quasiparticles. In our case those are the Bogoliubov quasiparticles. We introduce a distribution function of quasiparticles of the form

$$f^i_\sigma(\mathbf{k},t) = \langle \gamma^\dagger_{i\sigma}(\mathbf{k},t)\gamma_{i\sigma}(\mathbf{k},t)\rangle \;. \qquad (4.1)$$

For all our following considerations it is important to assume that the Bogoliubov particles constitute reasonably sharp quasiparticles. In equilibrium, *i.e.* in the absence of external perturbations (such as an applied voltage, temperature gradient, ...), the distribution function is given by familiar Fermi-Dirac distribution

$$f^i_\sigma(\mathbf{k},t) = n_f^0(\epsilon_i(\mathbf{k})) = \frac{1}{e^{\frac{\epsilon_i(\mathbf{k})}{T}}+1} \;, \qquad (4.2)$$

where $\epsilon_i(\mathbf{k})$ is given in Eq. (2.7).

In order to deal with all the Bogoliubov quasiparticles on equal footing, we introduce a local basis, where the node is again parametrized by i. We assume that we apply a temperature gradient across the system, such that the temperature at position $\mathbf{r}$ is given by $T(\mathbf{r}) = T+\mathbf{r}\cdot\nabla T$. The driving term assumes the generic form (we drop the spin-index for reasons of simplicity; it will trivially be accounted for by a factor of 2 in the end)

$$\begin{aligned} X_i \;&=\; \frac{\partial\epsilon_i}{\partial\mathbf{k}(\mathbf{k})}\frac{\nabla T}{T^2}\epsilon_i(\mathbf{k}) n_f^0(\epsilon_i(\mathbf{k}))n_f^0(-\epsilon_i(\mathbf{k})) \;, \\ \frac{\partial\epsilon_i(\mathbf{k})}{\partial\mathbf{k}} \;&=\; \frac{\left(\mathbf{v}_F^i(\mathbf{v}_F^i\cdot\mathbf{k})+\mathbf{v}_\Delta^i(\mathbf{v}_\Delta^i\cdot\mathbf{k})\right)}{\epsilon_i(\mathbf{k})} \;. \end{aligned} \qquad (4.3)$$

The form of Eq. (4.3) is markedly different from an isotropic system. In an isotropic system it is always possible to choose a basis such that one can formulate the problem in terms of the angle enclosed between ∇T and $\mathbf{k}$, *i.e.*, $X_i \propto v_F^2 |\nabla T| k \cos\left(\angle(\nabla T, \mathbf{k})\right)$[23,24]. This allows to find a much simpler solution than in our case, where the angular dependence has to be taken seriously. Our strategy to properly account for the anisotropy will be to consider two independent equations for the particles moving parallel and perpendicular to the nodal points.

The Boltzmann equation assumes the following form

$$\partial_t f^i(\mathbf{k},t) - X_i = -\mathcal{I}^i_{\rm coll} \; , \tag{4.4}$$

and since we are only interested in the time-independent solution we arrive at the following simple equation

$$X_i = \mathcal{I}^i_{\rm coll} \; , \tag{4.5}$$

which provides the basis for further discussions. It proves convenient to introduce the following simplifications at this point. We rescale momenta to eliminate all factors of v_F, v_Δ, and T, and go into a representation in polar coordinates, yielding the following compact form of the driving term

$$\begin{aligned} \tilde{X}_i &= -\left(\mathbf{v}^i_F \xi_i + \mathbf{v}^i_\Delta \Delta_i\right) k \frac{\nabla T}{T} n^0_f(k) n^0_f(-k) \\ &= \tilde{X}^F_i + \tilde{X}^\Delta_i \end{aligned} \tag{4.6}$$

where

$$\begin{aligned} \xi_i(\theta) &= = \cos\left(\theta + (i-1)\frac{\pi}{2}\right) \; , \\ \Delta_i(\theta) &= \sin\left(\theta + (i-1)\frac{\pi}{2}\right) \; . \end{aligned} \tag{4.7}$$

Also note that in the above expression the temperature has been scaled out of the Fermi function n^0_f, rendering all expressions dimensionless. The splitting of the driving term accounts for the fact that we finally solve two independent equations for the parallel and perpendicular electrons and is consistent with treatments using the Kubo formula[12].

The structure of the driving term motivates an ansatz for the solution of the Boltzmann transport equation given by

$$\begin{aligned} \delta\tilde{f}_i(k,\theta) &= \mathbf{v}^i_F \xi_i(\theta) \frac{\mathbf{v}^i_F \nabla T}{T} n^0_f(k) n^0_f(-k) \Psi^i_F(k,\theta,\overline{v}) \\ &+ \mathbf{v}^i_\Delta \Delta_i(\theta) \frac{\mathbf{v}^i_\Delta \nabla T}{T} n^0_f(k) n^0_f(-k) \Psi^i_\Delta(k,\theta,\overline{v}) \; , \end{aligned} \tag{4.8}$$

where the superscript i of the functions $\Psi^i_F(k,\theta,\overline{v})$ and $\Psi^i_\Delta(k,\theta,\overline{v})$, respectively, accounts for the fact that we can define a local basis for every Dirac point itself. Again, it is worthwhile to contrast this expression from an isotropic system like graphene. In such a system we would not need to distinguish the two different nodal directions and we could simply choose an ansatz of the form $\Psi(|\mathbf{k}|)\nabla T \cdot \mathbf{k}$ due to the spherical symmetry of the problem.

A few words on the symmetry of the obtained expressions are in order here, since this will help to simplify life a lot in the following. A refined discussion of the symmetries is presented in Appendix A. The symmetry of the driving term under exchange of vis-a-vis nodes, *i.e.* $X_i(k,\theta) = X_{i+2}(k,\theta)$, enforces $\delta\tilde{f}_i(k,\theta) = \delta\tilde{f}_{i+2}(k,\theta)$. Furthermore we can deduce from the symmetries of the driving term that

$$\Psi^i_{F/\Delta}(k,\theta,\overline{v}) = \Psi^i_{F/\Delta}(k,-\theta,\overline{v}) = \Psi^i_{F/\Delta}(k,\theta\pm\pi,\overline{v}) \; . \tag{4.9}$$

Following Eq. (3.1) we can derive an expression for the energy current carried by the Bogoliubov particles of the form (note that we have $N_f = 2$ in this expression)

$$\begin{aligned} \mathbf{j}_E &= \frac{2T^2}{v_F v_\Delta} \sum_{i=1}^{4} \int \frac{d\Omega_{\mathbf{k}} dk}{(2\pi)^2} k^2 \left[\mathbf{v}^i_F \xi_i + \mathbf{v}^i_\Delta \Delta_i\right] \delta\tilde{f}_i(k,\theta) \\ &= \frac{4T^2}{v_F v_\Delta} \sum_{i=1}^{2} \int \frac{d\Omega_{\mathbf{k}} dk}{(2\pi)^2} k^2 \left[\mathbf{v}^i_F \xi_i + \mathbf{v}^i_\Delta \Delta_i\right] \delta\tilde{f}_i(k,\theta) \; . \end{aligned} \tag{4.10}$$

Taking into account all the aforementioned symmetries of the problem we finally arrive at a relatively simple expression for the thermal current carried by the quasiparticles under an applied thermal gradient across the sample

$$\begin{aligned} \mathbf{j}_E &= \mathbf{V}_F \int \frac{d\Omega_{\mathbf{k}} dk}{(2\pi)^2} k^2 \xi^2_1(\theta) n^0_f(k) n^0_f(-k) \Psi^1_F(k,\theta,\overline{v}) \\ &+ \mathbf{V}_\Delta \int \frac{d\Omega_{\mathbf{k}} dk}{(2\pi)^2} k^2 \Delta^2_1(\theta) n^0_f(k) n^0_f(-k) \Psi^1_\Delta(k,\theta,\overline{v}) \; , \end{aligned} \tag{4.11}$$

where the terms mixing v_F and v_Δ vanish according to Eq. (4.9), due to

$$\int d\theta \Delta_i(\theta) \xi_i(\theta) \Psi^i_{F/\Delta}(k,\theta,\overline{v}) = 0 \; . \tag{4.12}$$

This is analogous to the vanishing of mixed terms in the treatment by Durst and Lee[12]. We furthermore introduced the following abbreviation:

$$\mathbf{V}_{F/\Delta} = \frac{4T^2}{v_F v_\Delta} \left(\mathbf{v}^1_{F/\Delta} \mathbf{v}^1_{F/\Delta} \nabla T + \mathbf{v}^2_{F/\Delta} \mathbf{v}^2_{F/\Delta} \nabla T\right) \; . \tag{4.13}$$

Since we are interested in the transport coefficient κ_{xx} (all others follow from symmetry) we can give a generic expression as

$$\begin{aligned} \kappa_{xx} &= -4T^2 \overline{v} \int \frac{d\Omega_{\mathbf{k}} dk}{(2\pi)^2} k^2 \xi^2_1(\theta) n^0_f(k) n^0_f(-k) \Psi^1_F(k,\theta,\overline{v}) \\ &- \frac{4T^2}{\overline{v}} \int \frac{d\Omega_{\mathbf{k}} dk}{(2\pi)^2} k^2 \Delta^2_1(\theta) n^0_f(k) n^0_f(-k) \Psi^1_\Delta(k,\theta,\overline{v}) \; . \end{aligned} \tag{4.14}$$

The collision integral $\mathcal{I}^i_{\rm coll}$ for a particle living at node i will in general be composed of two sources of relaxation, namely the contribution due to disorder, henceforth called $\mathcal{I}^i_{\rm dis}$, and the contribution due to the inelastic scattering, $\mathcal{I}^i_{\rm inel}$, *i.e.*,

$$\mathcal{I}^i_{\rm coll} = \mathcal{I}^i_{\rm dis} + \mathcal{I}^i_{\rm inel} \ . \tag{4.15}$$

The following sections are devoted to the understanding of the single and combined effect of those two scattering mechanisms.

A. Thermal transport in a disordered d-wave superconductor

Within this subsection we investigate the thermal transport in a system where the only relaxation mechanism is provided by dilute disorder. This has also been discussed in the thesis by Paaske[25] and simply serves as a reference. We introduce a parameter measuring the strength of impurity scattering

$$\alpha = \frac{2n_{\rm imp}\hat{u}^2}{v_F v_\Delta} \ , \tag{4.16}$$

where

$$\hat{u}^2 = \left(V^{ij}_{\mathbf{k}\mathbf{k}'}\right)^2 \ . \tag{4.17}$$

Expanding the collision term due to disorder to linear order in the deviation from equilibrium leaves us with the following expression

$$\mathcal{I}^i_{\rm dis} = \frac{\alpha k T}{4}\int \frac{d\Omega_{\mathbf{k}'}}{2\pi}\sum_{j=1}^{4} T^+_{ij}(\theta,\phi)\left[\delta\tilde{f}_i(k,\theta) - \delta\tilde{f}_j(k,\phi)\right] \ , \tag{4.18}$$

where we introduced the short-hand notation

$$T^\kappa_{ij}(\theta,\phi) = 1 + \kappa\xi_i(\theta)\xi_j(\phi) - \kappa\Delta_i(\theta)\Delta_j(\phi) \ , \tag{4.19}$$

with $\xi_i(\theta)$ and $\Delta_i(\theta)$ defined in Eq. (4.7) and $\kappa = \pm$. This factor simply accounts for the usual coherence factors[9]. We are thus left with the task of solving the equation

$$\tilde{X}_i = \mathcal{I}^i_{\rm dis} \ . \tag{4.20}$$

Plugging in the ansatz (4.8) we see that

$$\begin{aligned}\mathcal{I}^i_{\rm dis} &= \alpha k T\delta\tilde{f}_i(k,\theta) - \frac{\alpha k}{2}\sum_{j=1}^{2}\int\frac{d\Omega_{\mathbf{k}'}}{2\pi}\delta\tilde{f}_j(k,\phi) \\ &= \alpha k T\delta\tilde{f}_i(k,\theta) \ . \end{aligned} \tag{4.21}$$

where the second term in the first line vanishes due to the isotropic nature of the impurity scattering, which we assumed for reasons of simplicity. In the more generic case[25] this is no longer true. In our problem, however, the second term will always vanish also in the presence of the inelastic scattering by virtue of Eq. (4.9).

We can now find a simple solution given by

$$\Psi^i_F(k,\theta,\overline{v}) = \Psi^i_\Delta(k,\theta,\overline{v}) = \frac{1}{\alpha} \ . \tag{4.22}$$

Using Eq. (4.14) it is straightforward to arrive at an expression for the thermal transport coefficient, given by

$$\kappa_{xx} = \frac{\pi T}{12 n_{imp}\hat{u}^2}\left(v_F^2 + v_\Delta^2\right) \ , \tag{4.23}$$

which is the central result of this section. Paaske[25] showed that this discussion can easily be extended to include the case of anisotropic scattering, which means we are formally allowing for three different scattering matrix elements: intranodal scattering, which we denote $V^{11}_{\mathbf{k}\mathbf{k}'}$, scattering between adjacent nodes called $V^{12}_{\mathbf{k}\mathbf{k}'}$, and $V^{13}_{\mathbf{k}\mathbf{k}'}$ for scattering across the Brillouin zone. In this more general case the thermal conductivity reads

$$\kappa_{xx} = \frac{\pi T}{12 n_{imp} u_0^2}\left(\frac{v_F^2}{1-\delta} + \frac{v_\Delta^2}{1+\delta}\right) \ , \tag{4.24}$$

where

$$\begin{aligned} u_0^2 &= \frac{\left(V^{11}_{\mathbf{k}\mathbf{k}'}\right)^2 + \left(V^{13}_{\mathbf{k}\mathbf{k}'}\right)^2 + 2\left(V^{12}_{\mathbf{k}\mathbf{k}'}\right)^2}{4} \ , \\ \delta &= \frac{\left(V^{11}_{\mathbf{k}\mathbf{k}'}\right)^2 - \left(V^{13}_{\mathbf{k}\mathbf{k}'}\right)^2}{2u_0^2} \ . \end{aligned} \tag{4.25}$$

This expression nicely reduces to Eq. (4.23) for isotropic scattering, *i.e.*, $V^{11}_{\mathbf{k}\mathbf{k}'} = V^{12}_{\mathbf{k}\mathbf{k}'} = V^{13}_{\mathbf{k}\mathbf{k}'}$.

However, in the following sections we will concentrate on isotropic scattering, since we are interested in the interplay of disorder scattering and scattering from the nematic order parameter. Another comment on this result is in order here. In contrast to the universal conductivity[12,19] this result depends upon the impurity concentration explicitly, which also implies that the non-interacting problem in the clean limit has an infinite thermal conductivity. Experimental evidence, however, points towards the existence of a universal conductivity in the limit $T \to 0$ independent of the scattering strength[16,17,18]. The universal conductivity obtains in the limit when temperature T is much smaller than the impurity bandwidth γ, *i.e.* $T \ll \gamma$, see Ref. 19. Our calculation, however, addresses the opposite limit, such that this discrepancy does not constitute a problem and is furthermore consistent with Ref. 24. In the limit $T \ll \gamma$, the above treatment must be, in the spirit of a quantum Boltzmann equation[19], supplemented by a field renormalization stemming from the real part of the self-energy (this is neglected in the above Boltzmann equation, but can be incorporated in a straightforward manner[26,27]), which cancels the explicit dependence upon the impurity scattering. This establishes the equivalence with the Kubo-formula calculations employing a self consistent Born approximation[12], where the impurity scattering induces a finite density of states at the Fermi level, leading to the famous universal conductivity.

B. Thermal transport at the nematic transition

Within this section we consider the electronic scattering off the nematic mode. The central approximation in this section is to assume the bosonic sector to be in equilibrium, which implies that the bosonic system is not excited by the temperature gradient. One argument in favor of this point of view is to consider a situation in which the temperature gradient is applied along the v_F direction of the electrons at the nodal point denoted 1. This implies, that for the nodal particles living around nodes 2 and 4, only the electrons moving along the gap direction, v_Δ are excited, and thus in the limit of large anisotropy subdominant in their contribution to the thermal transport. The propagator of the bosonic mode is linked to contributions from all the fermionic nodes, and so its response to the thermal gradient is suppressed by a factor of $1/N_f$. A similar discussion has been carried out in the study of transport in bosonic theories in the large-N limit[14].

Another argument in favor of this approximation comes from rewriting the problem integrating out the bosons, which leads to a purely fermionic problem, see Appendix C. In the fermionic language one can clearly see that a scattering involving fermions at adjacent nodes can relax the thermal current, whereas scattering between electrons at the same node or nodes across the Brillouin zone cannot. This argument, however, only works for large anisotropy, *i.e.* $\overline{v} \gg 1$. Furthermore, we will show in Appendix C, that in the limit $\overline{v} = 1$, where we have a relativistically invariant theory, we recover an infinite thermal conductivity, as was found in graphene[23] and for generic relativistically invariant theories[6,54].

We first elaborate on the collision integral stemming from the scattering of the nodal fermions from the nematic order parameter fluctuations. It assumes the following generic form

$$\begin{aligned}\mathcal{I}^1_{\rm inel} &= \frac{T\lambda^2}{4v_F v_\Delta}\int \frac{d\Omega_{\mathbf{k}'}}{2\pi} dk' k' T^-_{11}(\theta,\phi) D''^{k-k'}_{\mathbf{k}-\mathbf{k}'} \left(\left(n_B(k-k') + n^0_f(-k')\right)\delta\tilde{f}_1(k,\theta) - \left(n_B(k-k') + n^0_f(k)\right)\delta\tilde{f}_1(k',\phi)\right) \\ &+ \frac{T\lambda^2}{4v_F v_\Delta}\int \frac{d\Omega_{\mathbf{k}'}}{2\pi} dk' k' T^-_{13}(\theta,\phi) D''^{k+k'}_{\mathbf{k}-\mathbf{k}'} \left(\left(n_B(k+k') + n^0_f(k')\right)\delta\tilde{f}_1(k,\theta) - \left(n_B(k+k') + n^0_f(k)\right)\delta\tilde{f}_1(k',\phi)\right) \ , \end{aligned} \tag{4.26}$$

which is a generalization of the expressions shown in Refs. 44,45 accounting for the coherence factors and the different nodes. Furthermore, we exploit the fact that $\delta\tilde{f}_i(k,\theta) = \delta\tilde{f}_{i+2}(k,\theta)$ and introduced $\tilde{D}''^{k-k'}_{\mathbf{k}-\mathbf{k}'}$ being the γ times the imaginary part of the retarded Green's function of the bosonic modes, which was introduced in its imaginary frequency form in Eq. (2.11). The full problem is analytically not tractable and has to be solved numerically. We thus use a variational approach[43,44,45,46], which allows to determine a bound for the conductivity. The interaction parameter λ drops out of the problem exactly which simpliy reflects the fact that the present perturbation theory is not controlled in the smallness of λ, but in the smallness of $1/N_f$. It is interesting to note that the functions $\Psi_{F/\Delta}(k,\theta,\overline{v})$ acquire a true angular dependence in contrast to the pure isotropic impurity scattering problem. The presence of the nematic mode leads to a non-trivial renormalization of the velocity-parameter $\overline{v}$, see Ref. 8, which will be taken into account later.

We set up the variational problem defining the appropriate matrix elements

$$
\begin{aligned}
\mathcal{Q} &= \mathcal{Q}_F + \mathcal{Q}_\Delta = \mathcal{X} - \mathcal{N} - \mathcal{D} \\
\mathcal{X} &= \frac{T}{\mathbf{v}_F^i \nabla T} \int \frac{kdkd\theta}{(2\pi)^2} \cos\theta \Psi_F^i(k,\theta,\overline{v}) \tilde{X}_i^F(k,\theta,\overline{v}) + \frac{T}{\mathbf{v}_\Delta^i \nabla T} \int \frac{kdkd\theta}{(2\pi)^2} \sin\theta \Psi_\Delta^i(k,\theta,\overline{v}) \tilde{X}_i^\Delta \\
\mathcal{N} &= \frac{4T}{N_f} \int \frac{kdkd\theta}{(2\pi)^2} \frac{d\phi}{2\pi} dk' k' T_{ii}^-(\theta,\phi) \tilde{D}''^{k-k'}_{\mathbf{k}-\mathbf{k}'} n_f^0(k) n_f^0(-k) \left(n_B(k-k') + n_f^0(-k')\right) \left(\cos\theta \Psi_F^i(k,\theta,\overline{v}) - \cos\phi \Psi_F^i(k',\phi,\overline{v})\right)^2 \\
&+ \frac{4T}{N_f} \int \frac{kdkd\theta}{(2\pi)^2} \frac{d\phi}{2\pi} dk' k' T_{ii}^-(\theta,\phi) \tilde{D}''^{k-k'}_{\mathbf{k}-\mathbf{k}'} n_f^0(k) n_f^0(-k) \left(n_B(k-k') + n_f^0(-k')\right) \left(\sin\theta \Psi_\Delta^i(k,\theta,\overline{v}) - \sin\phi \Psi_\Delta^i(k',\phi,\overline{v})\right)^2 \\
&+ \frac{4T}{N_f} \int \frac{kdkd\theta}{(2\pi)^2} \frac{d\phi}{2\pi} dk' k' T_{ii+2}^-(\theta,\phi) \tilde{D}''^{k+k'}_{\mathbf{k}-\mathbf{k}'} n_f^0(k) n_f^0(-k) \left(n_B(k+k') + n_f^0(k')\right) \left(\cos\theta \Psi_F^i(k,\theta,\overline{v}) - \cos\phi \Psi_F^i(k',\phi,\overline{v})\right)^2 \\
&+ \frac{4T}{N_f} \int \frac{kdkd\theta}{(2\pi)^2} \frac{d\phi}{2\pi} dk' k' T_{ii+2}^-(\theta,\phi) \tilde{D}''^{k+k'}_{\mathbf{k}-\mathbf{k}'} n_f^0(k) n_f^0(-k) \left(n_B(k+k') + n_f^0(k')\right) \left(\sin\theta \Psi_\Delta^i(k,\theta,\overline{v}) - \sin\phi \Psi_\Delta^i(k',\phi,\overline{v})\right)^2 \\
\mathcal{D} &= \frac{\alpha T}{2} \int \frac{kdkd\theta}{(2\pi)^2} k n_f^0(k) n_f^0(-k) \left(\cos\theta \Psi_F^i(k,\theta,\overline{v})\right)^2 + \frac{\alpha T}{2} \int \frac{kdkd\theta}{(2\pi)^2} k n_f^0(k) n_f^0(-k) \left(\sin\theta \Psi_\Delta^i(k,\theta,\overline{v})\right)^2 \ , \qquad (4.27)
\end{aligned}
$$

where $\mathcal{X}$ encodes the driving term, $\mathcal{N}$ denotes the scattering from the nematic mode, and $\mathcal{D}$ is simply the scattering from impurities. The above form is relatively simple and the reasons for its simplicity are explained in more detail in Appendix A in a general discussion of symmetries. In the following calculation we will assume $N_f = 2$. In this framework it is possible to obtain the Boltzmann transport equation by demanding a maximization of the functional in the following sense

$$
\frac{\partial \mathcal{Q}}{\partial \cos\theta \Psi_F^i(k,\theta,v)} = \frac{\partial \mathcal{Q}}{\partial \sin\theta \Psi_\Delta^i(k,\theta,v)} = 0 \ . \quad (4.28)
$$

By virtue of Eq. (4.12) the contribution associated with the direction perpendicular to the Fermi surface and the one associated with the parallel (gap) direction decouple nicely, and we are left with two equations that we can solve independently. Furthermore, comparing the expression for $\mathcal{X}$ in Eq. (4.27) and the expression for the transport coefficient κ_{xx} given in Eq. (4.14), it is obvious that we can extract the final solution by comparison. In order to solve the above integral equation we can now make a variational ansatz and maximize the above functional with respect to the coefficients. It seems sensible to choose the following ansatz

$$
\Psi_{F/\Delta}^i(k,\theta,\overline{v}) = -a_{F/\Delta}(\overline{v}) - b_{F/\Delta}(\overline{v}) \cos 2\theta \ , \quad (4.29)
$$

which again amounts to dealing with fermions perpendicular and parallel to the Fermi surface, separately.

Of course, the whole series $\cos 2n\theta$ with n being an integer is allowed for symmetry reasons. However, it is easy to check that only $n = 1$ contributes to the heat current, which is why we concentrate on this mode. We choose the mode with no k-dependence, since this is the mode associated with energy conservation in a clean system, see Appendix C. This mode in a clean system with a fully relativistic Hamiltonian is not relaxed and leads to an infinite thermal conductivity. One can of course improve upon the approximation by including more modes, which is however beyond the scope of this work, since we are mainly interested in qualitative features. It is important to note that the expressions for $\mathcal{X}$ and $\mathcal{D}$ can be calculated analytically, yielding

$$
\begin{aligned}
\mathcal{X} &= \frac{\pi}{24} \left(a_F + a_\Delta + \frac{b_F}{2} + \frac{b_\Delta}{2} \right) \\
\mathcal{D} &= \frac{\alpha T \pi}{48} \left(a_F^2 + a_\Delta^2 + \frac{b_F^2}{2} + \frac{b_\Delta^2}{2} + a_F b_F + a_\Delta b_\Delta \right) \ . \qquad (4.30)
\end{aligned}
$$

The thermal conductivity will finally be given by the simple expression

$$
\begin{aligned}
\frac{\kappa_{xx}}{T} &= \frac{\pi}{6} \overline{v} \left[a_F(\overline{v}) + \frac{b_F(\overline{v})}{2} \right] \\
&+ \frac{\pi}{6} \frac{1}{\overline{v}} \left[a_\Delta(\overline{v}) + \frac{b_\Delta(\overline{v})}{2} \right] \ . \qquad (4.31)
\end{aligned}
$$

This expression neglects the contribution of the effective bosonic modes to the heat conductivity. In Appendix D we further comment that fact.

V. THERMAL CONDUCTIVITY AT THE NEMATIC PHASE TRANSITION

We now turn our attention to the numerical solution of Eq. (4.31) in various situations.

In a first part, Sec. V A, we will analyze a clean system, in which the only current relaxation stems from inelastic scattering of the electronic quasiparticles from the effective bosonic mode. In a second step, Sec. V B, we will add dilute disorder to the problem and consider the full problem.

It was shown that the nematic phase transition is described by a fixed point located at infinite anisotropy, *i. e.* $v_F/v_\Delta \to \infty$[8] . This fixed point is approached in a

logarithmic manner upon lowering the temperature. This implies that in order to do a realistic calculation of the thermal conductivity one has to take into account the logarithmic flow of the velocity-anisotropy, which will lead to a logarithmic enhancement of the thermal conductivity.

We review the flow equations of the velocities under the renormalization group transformation in Appendix B to the extend needed for the calculations presented in this section.

All the plots presented in this section originate from a combination of Eq. (4.31) and the flow equations presented in Appendix B.

A. Quasiparticle thermal conductivity in a clean system

In this section we consider a clean system, in which the only source of thermal current relaxation stems from inelastic scattering. For a relativistically invariant theory, which is the case for $\overline{v} = 1$, an infinite thermal conductivity is to be expected[6,23,24,54]. However, in Appendix C we show that in an anisotropic system the zeromode of the isotropic system acquires a mass and is consequently relaxed.

Experimental evidence suggests a value of $v_F/v_\Delta \approx 20$. Our reference point in the following plot is thus given by this ratio; since we are only interested in the qualitative features the initial temperature of the integrated flow is chosen arbitrarily and called T_0. Fig. 1 contains a comparison of three curves in the clean limit. The uppermost curve is the full curve with all parameters flowing, *i.e.* it simply plots the numerical solution of Eq. (4.31). The middle curve is a plot of Eq. (4.31), where, in contrast to the upper curve, the temperature dependence of $a_{F/\Delta}$ and $b_{F/\Delta}$ is not taken into account. This is thus the curve which obtains if in the universal conductivity formula[12] the running velocity ratio is taken into account. The third curve simple serves as a reference showing an unrenormalized flat curve.

In the next section we will additionally consider the interplay between disorder and interaction with the bosonic mode.

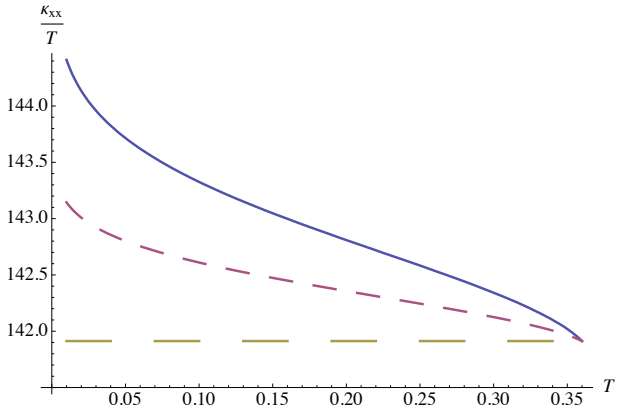

Figure 1: (Color online) The uppermost curve (solid/blue) shows the full numerical solution of Eq. (4.31), taking into account the temperature renormalization effect of the anisotropy ratio. The middle curve (short dashes/red) shows the universal limit conductivity with running couplings, whereas the lowest curve (long dashes/green) simply serves as a reference curve. All plots employ an initial anisotropy ratio $v_F/v_\Delta = 20$ at an arbitrary temperature T_0.

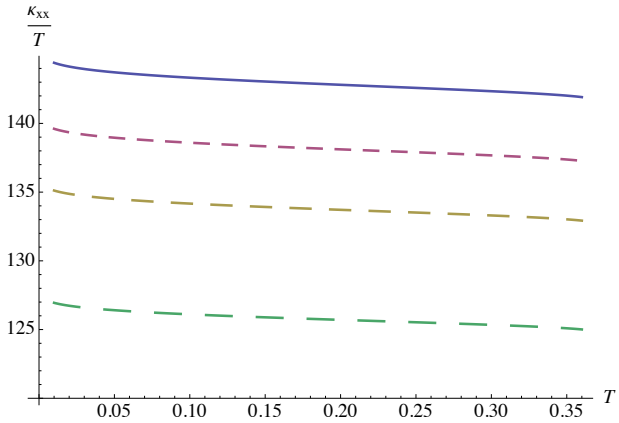

Figure 2: (Color online) This plot shows different impurity strengths in increasing order from top to bottom with the uppermost curve being for the clean system ($\alpha = 0$, $\alpha = 0.05$, $\alpha = 0.1$, $\alpha = 0.2$), *i.e.* κ_{xx}/T, for an initial anisotropy ratio $v_F/v_\Delta = 20$ at an arbitrary temperature T_0 for different disorder levels.

B. Quasiparticle thermal conductivity in a disordered system

As we showed in the discussion of the thermal conductivity in a disordered d-wave superconductor, the impurity scattering strength does not vanish if the field renormalization is not taken into account, see Sec. IV A. This of course has consequences if one considers the problem taking into account both scattering mechanisms, as shown in Fig. 2. We see that for different disorder levels, parametrized by α, the curve has different offsets, being maximal in the clean system, as one would expect. On top of this, by the definition of α given in Eq. (4.16), the disorder strength is flowing under the renormalization group transformation. This effect is illustrated in Fig. 3, where the upper curve takes into account the disorder renormalization, whereas the lower curve, for reference, neglects this effect.

VI. CONCLUSION

In this paper we have shown that at the nematic transition there is an enhancement of the thermal conductivity due to renormalization effects towards the fully

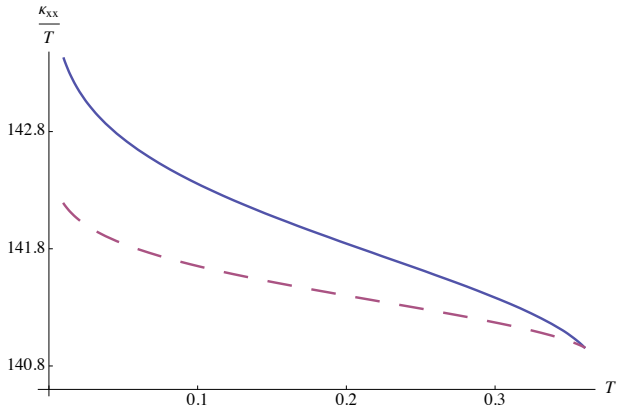

Figure 3: (Color online) This plot shows the effect of disorder renormalization (initial value $\alpha = 0.01$) due to the RG flow of the velocities. The upper curve (solid/blue) shows the full solution of Eq. (4.31) taking into account the disorder renormalization, whereas the lower curve (dashed/red) is plotted at fixed disorder, but everything else renormalized. Again, *i.e.* κ_{xx}/T is plotted for an initial anisotropy ratio $v_F/v_\Delta = 20$ at an arbitrary temperature T_0.

anisotropic RG fixed point at $(v_\Delta/v_F)^* = 0$. The central result of this paper is thus given by Eq. (4.31) and shown in Fig. 1. We addressed this experimentally relevant issue in the framework of the Boltzmann equation for the fermions, arguing that thermal transport of the collective bosonic excitation is suppressed in the large N_f limit[14]. Even though we have taken this limit, we believe that our treatment correctly captures the logarithmic enhancement as the central signature of the nematic transition. The full treatment including the bosonic mode is beyond the scope of this paper and requires a full discussion in the framework of coupled Keldysh equations or a treatment in the framework of the memory matrices[55].

Acknowledgments

We thank J. Paaske, M. Müller, Y. Huh, E.-A. Kim, M. Lawler, and B. Wunsch for useful discussions. This research was supported by the Deutsche Forschungsgemeinschaft under grant FR 2627/1-1 (LF), and by the NSF under grant DMR-0757145 (SS, LF).

Appendix A: SYMMETRIES OF THE DRIVING AND SCATTERING TERMS

Throughout the whole calculation we have made heavy use of symmetry properties of the present problem to simplify occurring expressions. In the derivation of the transport functional we have dealt with an expression of the following kind (we only keep angular variables at this point to simplify notation)

$$\sum_i \int d\theta d\phi T_{11}^-(\theta,\phi) D''(\theta,\phi) \left[\xi_i(\theta)\Psi_F^i(\theta)\right] \left[\xi_i(\theta)\Psi_F^i(\theta) + \Delta_i(\theta)\Psi_\Delta^i(\theta) - \xi_i(\phi)\Psi_F^i(\phi) - \Delta_i(\phi)\Psi_\Delta^i(\phi)\right]$$
$$+ \sum_i \int d\theta d\phi T_{13}^-(\theta,\phi) D''(\theta,\phi) \left[\xi_i(\theta)\Psi_F^i(\theta)\right] \left[\xi_i(\theta)\Psi_F^i(\theta) + \Delta_i(\theta)\Psi_\Delta^i(\theta) - \xi_i(\phi)\Psi_F^i(\phi) - \Delta_i(\phi)\Psi_\Delta^i(\phi)\right] . \quad \text{(A1)}$$

Taking a careful look at all the symmetries reveals that

$$\begin{aligned} T_{11}^-(\theta,\phi) &= T_{11}^-(-\theta,-\phi) = T_{11}^-(\theta\pm\pi,\phi\pm\pi) \\ T_{13}^-(\theta,\phi) &= T_{13}^-(-\theta,-\phi) = T_{13}^-(\theta\pm\pi,\phi\pm\pi) \\ D^{k-k'}(\theta,\phi) &= D^{k-k'}(-\theta,-\phi) = D^{k-k'}(\theta\pm\pi,\phi\pm\pi) . \end{aligned} \quad \text{(A2)}$$

This restricts a possible Fourier series of

$$G(\theta,\phi) = T_{11}^-(\theta,\phi) D^{k-k'}(\theta,\phi) + T_{13}^-(\theta,\phi) D^{k+k'}(\theta,\phi) \quad \text{(A3)}$$

to

$$\begin{aligned} G(\theta,\phi) &= \sum_{m,n} a_{2n}a_{2m}\cos 2n\theta\cos 2m\phi \\ &+ a_{2n+1}a_{2m+1}\cos(2n+1)\theta\cos(2m+1)\phi \\ &+ b_{2n+1}b_{2m+1}\sin(2n+1)\theta\sin(2m+1)\phi \\ &+ b_{2n}b_{2m}\sin 2n\theta\sin 2m\phi . \end{aligned} \quad \text{(A4)}$$

This property together with Eq. (4.9) implies that

$$\begin{aligned} &\int d\phi d\theta \xi_i(\theta)\Delta_i(\theta)\Psi_F^i(\theta)\Psi_\Delta^i(\theta)G(\theta,\phi) \\ &= \int d\phi d\theta \xi_i(\theta)\Delta_i(\phi)\Psi_F^i(\theta)\Psi_\Delta^i(\phi)G(\theta,\phi) = 0 . \end{aligned} \quad \text{(A5)}$$

This allows to rewrite Eq. (A1) as

$$\frac{1}{2} \quad \sum_i \int d\theta d\phi T_{11}^-(\theta,\phi) D''(\theta,\phi) \times$$
$$\times \left[\xi_i(\theta)\Psi_F^i(\theta) - \xi_i(\phi)\Psi_F^i(\phi)\right]^2$$
$$+ \frac{1}{2}\sum_i \int d\theta d\phi T_{13}^-(\theta,\phi) D''(\theta,\phi) \times$$
$$\times \left[\xi_i(\theta)\Psi_F^i(\theta) - \xi_i(\phi)\Psi_F^i(\phi)\right]^2 . \qquad \text{(A6)}$$

Appendix B: RG EQUATIONS

The full details of the derivation of the renormalization group equations were presented elsewhere[8], so we only repeat the crucial formulae. The methodology of the applied RG approach differs slightly from the more standard hard cutoff scheme, since a soft cutoff is more favorable in order to deal with the anisotropic velocities. Under the renormalization group transformation the fermionic velocities v_F and v_Δ modify according to

$$\frac{dv_F}{d\ell} = (C_1 - C_2) v_F \qquad \text{(B1)}$$

and

$$\frac{dv_\Delta}{d\ell} = (C_1 - C_3) v_\Delta . \qquad \text{(B2)}$$

This implies that the ratio of the two velocities scales like

$$\frac{d(v_\Delta/v_F)}{d\ell} = (C_2 - C_3)(v_\Delta/v_F) , \qquad \text{(B3)}$$

where the functions C_1, C_2, and C_3 are given by

$$C_1 = \frac{2\overline{v}^{-1}}{\pi^3 N_f} \int_{-\infty}^{\infty} dx \int_0^{2\pi} d\theta f^{+,-,-}(\theta,\overline{v}) \mathcal{G}(x,\theta)$$
$$C_2 = \frac{2\overline{v}^{-1}}{\pi^3 N_f} \int_{-\infty}^{\infty} dx \int_0^{2\pi} d\theta f^{-,+,-}(\theta,\overline{v}) \mathcal{G}(x,\theta)$$
$$C_3 = \frac{2\overline{v}^{-1}}{\pi^3 N_f} \int_{-\infty}^{\infty} dx \int_0^{2\pi} d\theta f^{+,+,-}(\theta,\overline{v}) \mathcal{G}(x,\theta) \qquad \text{(B4)}$$

Furthermore,

$$\mathcal{G}^{-1}(x,\theta) = \frac{x^2 + \sin^2\theta}{\sqrt{x^2 + \overline{v}^{-2}\cos^2\theta + \sin^2\theta}} + \frac{x^2 + \cos^2\theta}{\sqrt{x^2 + \cos^2\theta + \overline{v}^{-2}\sin^2\theta}} \qquad \text{(B5)}$$

is the ϕ propagator inverse and

$$f^{a,b,c}(\theta,\overline{v}) = \frac{(ax^2 + b\cos^2\theta + c\overline{v}^{-2}\sin^2\theta)}{(x^2 + \cos^2\theta + \overline{v}^{-2}\sin^2\theta)^2} \qquad \text{(B6)}$$

with $a, b, c = \pm$. This is the full set of equations required for the calculation of all running parameters used in Sec. IV B.

Appendix C: COMMENTS ON FINITE CONDUCTIVITIES IN IDEAL SYSTEMS

Naively, in a clean system neglecting Umklapp scattering one would expect infinite response of the system to small perturbations, *i.e.* infinite transport coefficients, if interactions are momentum-conserving.

If we consider a clean Fermi liquid with electron-electron interactions we observe the following. If we apply an electrical field across the system and forbid Umklapp scattering, no current is relaxed and the response is infinite. Physically, this is very intuitive, formally, however, this requires some work. Two complementary approaches are conveniently used to calculate transport coefficients. In the framework of linear response theory using the Kubo formula the use of a conserving approximation scheme is obligatory. This implies self-energies and vertex corrections are not independent of each other, but have to be chosen accordingly. This was shown by Yamada and Yoshida[33], and later by Rosch[34] who pointed out the importance of Umklapp scattering in the Bethe-Salpeter equation for the current-vertex in order to obtain a finite electrical conductivity in clean systems. On the level of a Boltzmann equation, this can be seen very naturally. The scattering integral for electron-electron interaction assumes the well-known schematic form[51]

$$\mathcal{I} = \int F\left[X_{\mathbf{k}} + X_{\mathbf{k_1}} - X_{\mathbf{k-q}} - X_{\mathbf{k_1+q}}\right] . \qquad \text{(C1)}$$

It is obvious from the above equation that the right hand side vanishes by choosing

$$X_{\mathbf{k}} = c\mathbf{k} , \qquad \text{(C2)}$$

which just restates the conservation of momentum. Since the solution of the Boltzmann equation requires the inversion of the scattering operator, there is no solution due to the existence of a mode with zero eigenvalue, a so-called zero-mode and usually associated with a conserved quantity (in this case momentum), which makes the matrix-inversion singular. In a simple application of a relaxation time approximation this point can easily be missed. However, the existence of a zero mode implies the existence of a mode, which cannot decay, *i.e.* the associated scattering time diverges, which also implies the conductivity as defined by the Drude[51] formula to diverge. One would expect the same kind of reasoning to apply for the thermal conductivity. That this, however, is not true was already point out by Langer in 1962[35], who argued that in the presence of electron-electron interactions the Wiedemann-Franz law should not generally apply. This was later also shown in Refs. 31,40,41. The reason for that is that in general electron-electron interactions can carry an energy current. Therefore, momentum conservation does not imply heat current conservation and a finite thermal transport time is possible in systems with Galilean invariance[36,37,38]. As an extreme example one might consider a Hubbard-Model in its Mott insulating

phase. In such a system, no charge is transported, however, heat can be transported by spin-waves. This is just to illustrate that heat and charge transport are not as intimately connected as one might think.

It is instructive to compare the above reasoning with a relativistically invariant electronic theory, such as the Dirac theory in the way it applies to intrinsic graphene in the clean limit. It was pointed out in Refs. 23,24 that the conductivity in such a system can be finite, *i.e.* a momentum conserving interaction can relax a current due to the special particle-hole structure of the Dirac Hamiltonian. However, this is not true for the thermal conductivity, which is infinite. This can be traced back to the conservation of the energy component of the momentum-energy tensor and it is a generic property of relativistically invariant theories without any sort of translational symmetry breaking[6,54].

This again can very nicely be seen in a Boltzmann transport approach. Here again, the collision integral assumes the generic form shown in Eq. (C1) and the existence of a zero-mode implies a diverging scattering time. Here, the roles with respect to the Fermi liquid are reversed and the zero-mode of the scattering integral is only excited by a temperature gradient, but not by an electrical field, hence the finite electrical conductivity.

We can now turn to our problem. In order to discuss the issue of a finite thermal conductivity in our model, it proves favorable to restate the transport problem in a completely fermionic language, which is achieved by integrating out the effective bosonic degrees of freedom. This leads to a long-range electron-electron interaction, in analogy to the model for interacting fermions which applies to graphene[23,24]. One can conclude the following. For no anisotropy, *i.e.* $\overline{v}=1$, the scattering integral posesses a zero-mode for heat transport, which implies a diverging thermal conductivity. However, for finite anisotropy, the zero-mode ceases to exist, which implies the decay of thermal current even in a clean system, leading to a finite heat conductivity. The full details of this reasoning can be found below.

1. Recovery of infinite thermal conductivity in the clean system in the isotropic case

In this section we want to reconcile our approach with the fact that in a relativistically invariant theory in a clean system an infinite heat conductivity is expected[6,23,24,54]. In order to address this question properly we restate the initial problem in terms of fermions only by integrating out the bosonic degrees of freedom. The problem now is equivalent to a set of Dirac fermions interacting via a long-range interaction, described by the following action

$$\begin{aligned}\mathcal{S}_{\rm int} \;\propto\; & T^3\sum_{\omega_1,\omega_2,\nu}\int_{\mathbf{k},\mathbf{k}',\mathbf{q}}\sum_{a,b=1,2}\sum_{\sigma=1}^{N} D(\nu,\mathbf{q})\times\\ & \Psi^\dagger_{a,\mathbf{k}+\mathbf{q},\sigma}(\omega_1+\nu)\tau^x\Psi_{a,\mathbf{k},\sigma}(\omega_1)\times\\ & \times\;\Psi^\dagger_{b,\mathbf{k}'-\mathbf{q},\sigma}(\omega_2-\nu)\tau^x\Psi_{b,\mathbf{k}',\sigma}(\omega_2)\;. \end{aligned}\tag{C3}$$

We will sketch the derivation of the kinetic equation, which has been presented elsewhere[23,56]. The starting point of our discussion is given by the kinetic equation[56], which has to be generalized to incorporate a possible matrix structure of the Green's function (in our case the Green's function lives in spinor space within a structure due to the N_f spin and the nodal index, and thus has a $2N_f\times 2N_f$ structure)

$$\begin{aligned}& \left[\partial_T-\nabla_{\mathbf{R}}U(\mathbf{R},T)\nabla_{\mathbf{k}}\right]\mathbf{G}^{a<}(\mathbf{k},\omega;\mathbf{R},T)=\\ -\;& \mathbf{G}^{a<}(\mathbf{k},\omega;\mathbf{R},T)\mathbf{\Sigma}^{a>}(\mathbf{k},\omega;\mathbf{R},T)\\ +\;& \mathbf{G}^{a>}(\mathbf{k},\omega;\mathbf{R},T)\mathbf{\Sigma}^{a<}(\mathbf{k},\omega;\mathbf{R},T)\end{aligned}\tag{C4}$$

The self-energy diagrams are the RPA-type contribution and the maximally crossed diagram[56]. This set of diagrams is commonly called the second order Born approximation. In a large-N_f framework the contribution of the maximally crossed diagram will be suppressed by a relative factor of $1/N_f$ with respect to the RPA-term and will consequently be dropped. In a next step we find that the Fourier transform with respect to the relative coordinates reads

$$\begin{aligned}\mathbf{\Sigma}^{a>,<}_{\alpha\beta}(\mathbf{k},\omega;T) \;=\; & \int\frac{d^2\mathbf{k_1}}{(2\pi)^2}\frac{d\omega_1}{2\pi}\frac{d^2\mathbf{k_2}}{(2\pi)^2}\frac{d\omega_2}{2\pi}\frac{d^2\mathbf{k_3}}{(2\pi)^2}\frac{d\omega_3}{2\pi}(2\pi)^3\delta(\mathbf{k}+\mathbf{k_1}-\mathbf{k_2}-\mathbf{k_3})\delta(\omega+\omega_1-\omega_2-\omega_3)\times\\ \times\;& \sum_b\Big[|D^R(\omega-\omega_2,\mathbf{k}-\mathbf{k_2})|^2\mathbf{G}^{b<,>}_{\gamma\delta}(\mathbf{k_1},\omega_1)\mathbf{G}^{a>,<}_{\alpha\beta}(\mathbf{k_2},\omega_2)\mathbf{G}^{b>,<}_{\delta\gamma}(\mathbf{k_3},\omega_3)\Big]\;.\end{aligned}\tag{C5}$$

We transform the above expression into the appropriate Bogoliubov quasiparticle basis of the γ_i through

$$\mathbf{G}^{a<,>}(\mathbf{k},\omega)=U^{-1}_{a\mathbf{k}}g^{a<,>}(\mathbf{k},\omega)U_{a\mathbf{k}}$$

where the unitary matrix $U^{-1}_{\mathbf{k}}$ was introduced in Eq. (2.6).

Accounting for the fact that the operators γ describe sharp quasiparticles the lesser and greater Green's functions are given by

$$g_\lambda^{a<}(\mathbf{k},\omega;T) = 2\pi\delta(\omega-\epsilon_\lambda^a(\mathbf{k},T))f_\lambda(\mathbf{k},T)$$

and

$$g_\lambda^{a>}(\mathbf{k},\omega;T) = 2\pi\delta(\omega-\epsilon_\lambda^a(\mathbf{k},T))\left[1-f_\lambda(\mathbf{k},T)\right] ,$$

where we assumed the distribution function of the quasiparticles to have no off-diagonal components, which is justified to linear order in the temperature gradient. Furthermore, we introduced the following convention

$$\epsilon_\lambda^a(\mathbf{k}) = \begin{cases} \epsilon^a(\mathbf{k}) & \lambda = 1,3 \\ -\epsilon^a(\mathbf{k}) & \lambda = 2,4 \end{cases} . \tag{C6}$$

The diagonal part of the distribution function using

$$M^a_{\lambda\lambda_1}(\mathbf{k},\mathbf{k_1}) = \left[U_{a\mathbf{k}}\tau^x U^{-1}_{a\mathbf{k_1}}\right]_{\lambda\lambda_1} \tag{C7}$$

reads

$$\begin{aligned}
&\left[\partial_T - \nabla_{\mathbf{R}}U(\mathbf{R},T)\nabla_{\mathbf{k}}\right]f_\mu(\mathbf{k},T) = 2\pi N\sum_b\int\frac{d^2\mathbf{k_1}}{(2\pi)^2}\frac{d^2\mathbf{q}}{(2\pi)^2}\delta(\epsilon_\mu^a(\mathbf{k})+\epsilon_\lambda^b(\mathbf{k}_1)-\epsilon_{\lambda_1}^a(\mathbf{k}+\mathbf{q})-\epsilon_{\lambda_2}^b(\mathbf{k_1}-\mathbf{q})) \\
&\times\ \left[|D^R(\epsilon_\mu^a(\mathbf{k})-\epsilon_{\lambda_1}^a(\mathbf{k}+\mathbf{q}),\mathbf{q})|^2 M_{\lambda_2\lambda}(\mathbf{k_1}-\mathbf{q},\mathbf{k_1})M_{\lambda\lambda_2}(\mathbf{k_1},\mathbf{k_1}-\mathbf{q})M_{\mu\lambda_1}(\mathbf{k},\mathbf{q}+\mathbf{k})M_{\lambda_1\mu}(\mathbf{q}+\mathbf{k},\mathbf{k})\right]\times \\
&\times\ \left[(1-f_\mu(\mathbf{k},T))(1-f_\lambda(\mathbf{k_1},T))f_{\lambda_1}(\mathbf{q}+\mathbf{k},T)f_{\lambda_2}(\mathbf{k_1}-\mathbf{q},T)\right. \\
&-\ \left. f_\mu(\mathbf{k},T)f_\lambda(\mathbf{k_1},T)(1-f_{\lambda_1}(\mathbf{q}+\mathbf{k},T))(1-f_{\lambda_2}(\mathbf{k_1}-\mathbf{q},T))\right] .
\end{aligned} \tag{C8}$$

It is interesting to note that in the case of isotropic velocities there is no need to distinguish the energy functions ϵ^a and ϵ^b any more, and the whole discussion of transport coefficients parallels the discussion in graphene, see Ref. 23. In this case all the observations made in graphene, including the presence of a collinear forward scattering divergence, hold. The only modification stems form the fact that the interaction is different, giving another numerical value of the universal prefactor. Furthermore, in this case the thermal conductivity is infinite due to the presence of a momentum mode, which is unaltered by the present of the electron-electron interaction. The situation changes in the case of anisotropic velocities. In this case it is obvious that we have two qualitatively different scattering processes. Here, for the term scattering from a to b with $a = b$ it is still possible to find a zero mode, since we can always rescale the moment in the local nodal basis, leading to an expression as in graphene, however with a different global prefactor. This is not true for the term scattering a to b with $b \neq a$. This term does not allow for such a simple rescaling operation and the vanishing of the zero mode is thus responsible for a non-infinite thermal conductivity. It is thus to be expected that in a clean system with a velocity anisotropy we find a finite thermal conductivity. This comes about naturally in the case where we consider electronic scattering off the effective bosonic propagator.

We can make the above stated more substantial by considering the ansatz introduced in Eq. (4.8), where we choose a constant function $\Psi_F^i(k,\theta,\overline{v}) = \Psi_\Delta^i(k,\theta,\overline{v}) = \text{const}$. We can write the linearized version as

$$\begin{aligned}
\mathcal{I}\ \propto\ &\left[\mathbf{v}_F^a\nabla T(\mathbf{v}_F^a\cdot\mathbf{k}) + \mathbf{v}_F^b\nabla T(\mathbf{v}_F^b\cdot\mathbf{k}') - \mathbf{v}_F^a\nabla T(\mathbf{v}_F^a\cdot(\mathbf{k}+\mathbf{q})) - \mathbf{v}_F^b\nabla T(\mathbf{v}_F^b\cdot(\mathbf{k}'-\mathbf{q}))\right. \\
+\ &\left.\mathbf{v}_\Delta^a\nabla T(\mathbf{v}_\Delta^a\cdot\mathbf{k}) + \mathbf{v}_\Delta^b\nabla T(\mathbf{v}_\Delta^b\cdot\mathbf{k}') - \mathbf{v}_\Delta^a\nabla T(\mathbf{v}_\Delta^a\cdot(\mathbf{k}+\mathbf{q})) - \mathbf{v}_\Delta^b\nabla T(\mathbf{v}_\Delta^b\cdot(\mathbf{k}'-\mathbf{q}))\right] .
\end{aligned} \tag{C9}$$

This expression generically vanishes for any anisotropy as long as $a = b$. If $a \neq b$, however, we can rewrite the above expression to yield

$$\begin{aligned}
\mathcal{I}\ \propto\ &\mathbf{v}_F^a\nabla T(\mathbf{v}_F^a\cdot\mathbf{q})\left(\overline{v}^{-2}-1\right) \\
+\ &\mathbf{v}_\Delta^a\nabla T(\mathbf{v}_\Delta^a\cdot\mathbf{q})\left(\overline{v}^2-1\right)
\end{aligned} \tag{C10}$$

This nicely restates the fact that the momentum mode is a zero mode of the problem for the isotropic system, *i.e.* $\overline{v} = 1$, whereas the mode becomes massive once $\overline{v}^2 \neq 1$. The vanishing of this zero mode seems to justify the existence of a finite thermal conductivity even in a clean system, provided the anisotropy is large, see Sec. IV B. The analysis of a clean electronic system and the interaction mediated thermal conductivity was carried out in Refs. 39,40, and redone in a decoupling scheme in bosonic fields by Catelani and Aleiner[41].

Appendix D: CONTRIBUTION OF THE EFFECTIVE BOSONIC MODE TO THE HEAT CURRENT

The full action of our problem at hand is given by the electronic d-wave superconductor coupled to the bosonic nematic mode, both of which can carry heat current. In a generic electron-phonon system the complete expression for the heat operator was given by Vilenkin *et al.*[28]. Our problem, however, is different in the sense, that the effective dynamics of the bosonic mode is created by the electrons, see Eq. (2.11). This is very similar to problems studied in the context of slave particle theories for the t-J model or in composite fermion theories of the fractional quantum hall effect, where an effective action for a U(1)-gauge field is generated and the analysis of transport quantities is tedious, see Refs. 47,48,49,50,53.

In order to derive an expression for the effective heat current carried by the bosonic mode we expand the saddle point action to third order following a minimal coupling scheme explained in Refs. 29,31,32. In the following we assume a thermal gradient in the direction of the Fermi-velocity at node 1, without loss of generality. This implies we have a thermal gauge field without a y-component in the local basis defined at node 1. Furthermore, we will later make the approximation that $\overline{v} \gg 1$, which implies that the thermal gradient decouples from the second term in Eq.(2.1). We thus find the minimally coupled version of the problem to be given by

$$\begin{aligned}
\mathcal{S} \;=\; & \int d\tau \sum_{\mathbf{k},\mathbf{k}',\sigma} \Psi^{\dagger}_{1\mathbf{k}\sigma} \begin{pmatrix} \partial_\tau - v_F(k_x\delta_{\mathbf{k},\mathbf{k}'} - iA_2^x(\mathbf{k}-\mathbf{k}')i\overleftrightarrow{\partial}_\tau) & v_\Delta k_y \delta_{\mathbf{k},\mathbf{k}'} \\ v_\Delta k_y \delta_{\mathbf{k},\mathbf{k}'} & \partial_\tau + v_F(k_x\delta_{\mathbf{k},\mathbf{k}'} - iA_2^x(\mathbf{k}-\mathbf{k}')i\overleftrightarrow{\partial}_\tau) \end{pmatrix} \Psi_{1\mathbf{k}'\sigma} \\
+ & \int d\tau \sum_{\mathbf{k},\mathbf{k}',\sigma} \Psi^{\dagger}_{2\mathbf{k}\sigma} \begin{pmatrix} \partial_\tau - v_F k_y \delta_{\mathbf{k},\mathbf{k}'} & v_\Delta(k_x\delta_{\mathbf{k},\mathbf{k}'} - iA_2^x(\mathbf{k}-\mathbf{k}')i\overleftrightarrow{\partial}_\tau) \\ v_\Delta(k_x\delta_{\mathbf{k},\mathbf{k}'} - iA_2^x(\mathbf{k}-\mathbf{k}')i\overleftrightarrow{\partial}_\tau) & \partial_\tau + v_F k_y \delta_{\mathbf{k},\mathbf{k}'} \end{pmatrix} \Psi_{2\mathbf{k}'\sigma} \,, \qquad \text{(D1)}
\end{aligned}$$

where $\overleftrightarrow{\partial}_\tau = \frac{1}{2}\left(\overrightarrow{\partial}_\tau - \overleftarrow{\partial}_\tau\right)$, see Ref. 31. We can proceed with the derivation of the effective heat vertex of the bosonic mode, which is achieved calculating the diagram shown in Fig. 4 a.). The "heat-vertex" with incoming frequency Ω_n and the external momentum $\mathbf{q}$ of the gauge field equal to zero, *i.e.* $\mathbf{q} = \mathbf{0}$, equates in the limit of zero temperature to

$$\begin{aligned}
\Gamma(\nu_n, \mathbf{k}, \Omega_n, \mathbf{q}=\mathbf{0}) \;=\; & \frac{\overline{v} N_f k_x}{16\Omega_n}\left(\frac{v_F^2k_x^2 + (\nu_n+\Omega_n)^2}{\sqrt{v_F^2k_x^2 + v_\Delta^2 k_y^2 + (\nu_n+\Omega_n)^2}} - \frac{v_F^2k_x^2 + \nu_n^2}{\sqrt{v_F^2k_x^2 + v_\Delta^2 k_y^2 + \nu_n^2}}\right) \\
+ & \frac{N_f k_x}{16\Omega_n \overline{v}}\left(\frac{v_F^2k_y^2 + \nu_n(\nu_n+\Omega_n)}{\sqrt{v_\Delta^2k_x^2 + v_F^2 k_y^2 + \nu_n^2}} - \frac{v_F^2k_y^2 + \nu_n(\nu_n+\Omega_n)}{\sqrt{v_\Delta^2k_x^2 + v_F^2 k_y^2 + (\nu_n+\Omega_n)^2}}\right) \\
\lim_{\overline{v}\gg 1} \;\approx\; & \frac{\overline{v} N_f k_x}{16\Omega_n}\left(\sqrt{v_F^2k_x^2 + (\nu_n+\Omega_n)^2} - \sqrt{v_F^2k_x^2 + \nu_n^2}\right) . \qquad \text{(D2)}
\end{aligned}$$

The derivation of this quantity is very tedious and involves the usage of Feyman parameters, see *e.g.* Ref. 42. However, it is interesting to note that the above expression, before taking the limit $1/\overline{v} \to 0$ in the last line, is *exact*. With the effective "heat-vertex" at hand we can proceed to calculate the bosonic contribution to the thermal transport. The corresponding diagrammatic expressions assume the form shown in Fig. 4 b.). At this stage it is very important to note that the above expression is even under $\nu_n \to -\nu_n$, $\Omega_n \to -\Omega_n$, and $\mathbf{k} \to -\mathbf{k}$. This is very important, since it implies that the heat-

current-heat-current correlation function is given by

$$\begin{aligned}
&\mathcal{K}(\Omega_n, \mathbf{q}=\mathbf{0}) \propto (\overline{v}N_f)^2\, T\sum_{\nu_n}\int \frac{d^2\mathbf{k}}{(2\pi^2)} \\
&\Gamma(\nu_n,\mathbf{k},\Omega_n,\mathbf{0})\left(\Gamma(\nu_n,\mathbf{k},\Omega_n,\mathbf{0})+\Gamma(-\nu_n,-\mathbf{k},-\Omega_n,\mathbf{0})\right) \\
&G_\phi(\mathbf{k},\nu_n+\Omega_n)G_\phi(\mathbf{k},\nu_n) \\
=\;& 2\,(\overline{v}N_f)^2\, T\sum_{\nu_n}\int \frac{d^2\mathbf{k}}{(2\pi^2)} \\
&\Gamma(\nu_n,\mathbf{k},\Omega_n,\mathbf{0})^2 G_\phi(\mathbf{k},\nu_n+\Omega_n)G_\phi(\mathbf{k},\nu_n)\;. \qquad \text{(D3)}
\end{aligned}$$

One comment has to be made at this stage, which is, that the heat-vertex in our case has been evaluated using the free Bogoliubov quasiparticle propagators. This, in general, constitutes a problem, once the d.c. limit in a response function is taken, since the resulting integrals are ill-defined and lead to infinite response coefficients[51]. This is also reflected in the large-N calculation in a fermion-gauge field system as they often appear in gauge theory descriptions of strongly interacting electronic systems of Ref. 49, where the calculation becomes invalid in the low-frequency limit and the temperature has to serve as a cutoff. It is an interesting aside to mention that the involved terms agree with the conserving approximation[33], however lacking the self-consistency. Starting from a heat-vertex calculated from free fermionic propagators, a calculation of the zero-frequency limit of the bosonic contribution to the thermal conductivity using the appropriate propagator for $\overline{v} \gg 1$ introduced in Eq. (2.13) following the prescription given in Ref. 32 does not yield a finite value. This can be traced back to the contribution of the free fermions and is an artifact of the neglect of the self-energies[33]. We have not explicitly performed the calculation taking into account the self-energies. It is, however, straightforward using a spectral representation of the electronic contributions and introducing a thermal broadening[52]. The important point is that the contribution of the bosonic modes to the thermal conductivity is finite and naively proportional to $\overline{v}^0$, which can easily be checked by taking the anisotropic bosonic propagator (Eq. (2.13)) and scaling all the momenta such as to make the resulting integral dimensionless. In this sense the bosonic contribution is down by $1/\overline{v}$ with respect to the fermionic contribution steming from the nodal fermions which move in perpendicular direction to the Fermi surface, compare Eq. (4.14) in Sec. IV. However, it is unclear whether this power-counting argument applies once the self-energies are taken into account. This question is postponed to later works. For a perfectly isotropic situation an analysis of the transport properties in terms of the fermionic model seems more appropriate, see Appendix C, leading to an infinite thermal conductivity in the absence of disorder. This is an important consistency check for any Kubo-formula based calculation of the thermal conductivity. In the isotropic limit one has to obtain an infinite thermal conductivity in the isotropic limit, comparable to an infinite electrical conductivity in a clean Fermi-liquid, which dictates

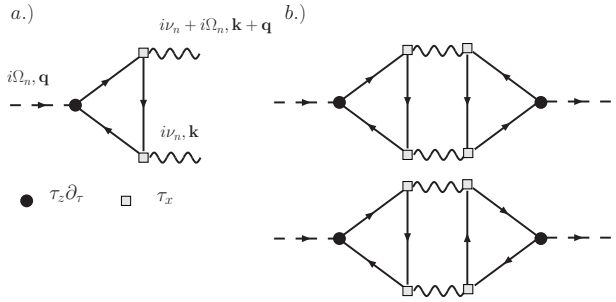

Figure 4: the effective heat vertex proportional to $\overline{v}$ according to Eq. (D2) is shown in a.); b.) shows the bosonic contribution to the thermal current.

the self-energy and vertex diagrams in the conserving approximation[33].

[1] S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature 393, 550 (1998).
[2] K. B. Cooper, M. P. Lilly, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. B 65, 241313 (2002).
[3] R. A. Borzi, S. A. Grigera, J. Farrell, R. S. Perry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, and A. P. Mackenzie, Science 315, 214 (2007).
[4] Y. Ando, K. Segawa, S. Komiya, and A. N. Lavrov, Phys. Rev. Lett. **88**, 137005 (2002).
[5] V. Hinkov , D. Haug, B. Fauque, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin and B. Keimer, Science, **319**, 597 (2008).
[6] M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. 85, 4940 (2000); *ibid.* 100, 089904(E) (2008); Int. J. Mod. Phys. B 14, 3719 (2000).
[7] E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, and S. A. Kivelson, Phys. Rev. B **77**, 184514 (2008).
[8] Y. Huh and S. Sachdev, Phys. Rev. B **78**, 064512 (2008)
[9] J. R. Schrieffer, *Theory of Superconductivity*, ().
[10] E. Fradkin, *Field Theories of Condensed Matter Systems*, (Addison-Wesley, Redwood City, 1991).
[11] A. Auerbach, *Interacting Electrons and Quantum Magnetism*, (Springer-Verlag, NewYork, 1994).
[12] A. C. Durst and P. Lee, Phys. Rev. B 62, 1270 (2000).
[13] L. Dell'Anna, M. Fabrizio, and C. Castellani, J. Stat. Mech. P02014 (2007)
[14] S. Sachdev, *Quantum Phase Transitions*, (Cambridge Univ. Press, Cambridge, 1999).
[15] K. Damle and S. Sachdev, Phys. Rev. B 56, 8714 (1997).
[16] L. Taillefer, B. Lussier, R. Gagnon, K. Behnia and H. Aubin, Phys. Rev. Lett. 79, 483 (1997).
[17] S. Nakamae, K. Behnia, L. Balicas, F. Rullier-Albenque, H. Berger, and T. Tamegai, Phys. Rev. B 63, 184509 (2001).
[18] M. Suzuki, M. A. Tanatar, N. Kikugawa, Z. Q. Mao, Y.

Maeno, and T. Ishiguro1, Phys. Rev. Lett. 88, 227004 (2002).
[19] M. J. Graf, S.-K. Yip, J. A. Sauls, and D. Rainer, Phys. Rev. B 53, 15147 (1996).
[20] A. C. Durst and S. Sachdev, arXiv:0810.3914 (2008).
[21] S. Sachdev, Phys. Rev. B 57, 7157 (1998).
[22] T. Senthil, M. Vojta, and S. Sachdev, Phys. Rev. B 69, 035111 (2004).
[23] L. Fritz, J. Schmalian, M. Müller, and S. Sachdev, Phys. Rev. B 78, 085416 (2008).
[24] M. Müller, L. Fritz, and S. Sachdev, arXiv:0805.1413 (2008).
[25] Jens Paaske, Ph.D. Thesis (1999), University of Copenhagen Ph.D. series 2000-01ISSN 0906-0286; http://www2.nbi.ku.dk/side105168.htm .
[26] J. Rammer and H. Smith, Rev. Mod. Phys. 58, 323 (1986).
[27] R. E. Prange and L. P. Kadanoff, Phys. Rev. 134, A566 (1964).
[28] A. Vilenkin and P. L. Taylor, Phys. Rev. B 18, 5280 (1978).
[29] A. I. Larkin and A. A. Varlamov, *Theory of fluctuations in superconductors*, (2005, Oxford University Press).
[30] A. A. Nersesyan, A. M. Tsvelik, and F. Wenger, Nucl. Phys. B 438, 561 (1995).
[31] J. Moreno and P. Coleman, cond-mat/9603079.
[32] A. DelMaestro, B. Rosenow, and S. Sachdev, arXiv:0807.2873.
[33] K. Yamada and K. Yoshida, Prog. Theor. Phys. 76, 621 (1986).
[34] A. Rosch, Ann. Phys., 15, 526 (2006).
[35] J. S. Langer, Phys. Rev. 128, 110 (1962).
[36] V. Ambegaoker and L. Tewordt, Phys. Rev. 134, A805 (1964).
[37] G. M. Eliashberg, Zh. Eksp. Teor. Fiz. 61, 125 (1971); JETP 34, 668 (1972).
[38] J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. 113, 982 (1959).
[39] A. A. Abrikosov and I. M. Khalatnikov, Rept. Prog. Phys. 22, 329 (1959).
[40] A.O. Lyakhov and E.G. Mishchenko, Phys. Rev. B. 67, 041304 (2003).
[41] G. Catelani and I. L. Aleiner, JETP 100, 331 (2005).
[42] M. E. Peskin and D. V. Schroeder, *An introduction to Quantum Field Theory*, (1995, Addison-Wesley Advanced Book Program).
[43] J. M. Ziman, *Electrons and Phonons*, Oxford University Press, Oxford (1960), Chapter 7.
[44] R. Hlubina and T. M. Rice, Phys. Rev. B 51, 9253 (1995).
[45] A. Rosch, Phys. Rev. Lett. 82, 4280 (1999); Phys. Rev. B 62, 4945 (2000).
[46] P. Arnold, G. D. Moore, L. G. Yaffe, JHEP **11**, 001 (2000).
[47] L. B. Ioffe and A. I. Larkin, Phys. Rev. B 39, 8988 - 8999 (1989).
[48] P. A. Lee and N. Nagaosa, Phys. Rev. B 46, 5621 (1992).
[49] Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, Phys. Rev. B 50, 17917 (1994).
[50] Y. B. Kim, P. A. Lee, and X.-G. Wen, Phys. Rev. B 52, 17275 (1995).
[51] G. D. Mahan, *Many Particle Physics*, (Springer, 3rd Edition, 2007).
[52] G. Aarts and J. M. Matrinez Resco, JHEP 204, 53 (2002).
[53] T. Senthil, M.Vojta, and S. Sachdev, Phys. Rev. B 69, 035111 (2004).
[54] S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Phys. Rev. B 76, 144502 (2007).
[55] D. Forster, *Hydrodynamic Fluctuations, Broken Symmetry, And Correlation Functions*, Westview Press (1995).
[56] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962) .