

# Absence of U(1) spin liquids in two dimensions

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(Dated: June 18, 2003)

## Abstract

Many popular models of fractionalized spin liquids contain neutral fermionic spinon excitations on a Fermi surface, carrying unit charges under a compact U(1) gauge force. We argue that instanton effects generically render such states unstable to confinement in two spatial dimensions, so that all elementary excitations are gauge neutral, and there is no spinon Fermi surface. Similar results are expected to apply to SU(2) spin liquids. However, fractionalized states can appear when the gauge symmetry is broken down to a discrete subgroup by the Higgs mechanism. Our argument generalizes earlier results on confinement in the pure gauge theory, and on the instability of the U(1) ‘staggered flux’ and ‘algebraic’ spin liquids with a Dirac spectrum for fermionic spinons.

## I. INTRODUCTION

The anomalous normal state of the cuprate superconductors has focused a great deal of attention on exotic states with electron fractionalization. One model, which has enjoyed a great deal of popularity [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23], has the electron spin reside on neutral  $S = 1/2$  fermionic ‘spinons’ which form a Fermi surface. The microscopic theory of the formation of spinons shows that each spinon carries unit charge of a compact gauge group, which is usually [2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]  $U(1)$ ; however, somewhat more involved theories with a  $SU(2)$  gauge group have also been considered [3, 16, 19, 22, 23]. The spinons interact with each other, and also with other possible ‘holon’ excitations, via exchange of the gauge boson. We will focus here on the  $U(1)$  case, but our arguments appear to have a direct generalization to the  $SU(2)$  case.

Polyakov [24] argued many years ago that the pure compact  $U(1)$  gauge theory is always confining at zero temperature in two spatial dimensions. In other words, there is no ‘Coulomb’ phase with a massless ‘photon’ excitation, and oppositely charged static test particles experience a confining potential which increases linearly with the distance between them. The confinement was induced by the proliferation of ‘instanton’ tunneling events. Given this fundamental result, one might wonder if similar effects are important in the models of electron fractionalization [2]. Indeed, in theories of quantum antiferromagnets with collinear spin correlations, such effects were crucial in disrupting possible spin liquid states and leading to ground states with bond order and confined spinons [25].

Many models of spin liquids have gapless fermionic spinon excitations, and in these cases the argument for confinement by proliferation of instantons is more delicate. The ‘staggered flux’ and related ‘algebraic spin liquids’ possess spinons with a relativistic Dirac-like spectrum, and consequently the instantons interact with an action which depends logarithmically on their spacetime separation [26]. By examining the renormalization of the fugacity of a single instanton, several authors [16, 26, 27] have argued that this logarithmic coupling leads to a regime in which instantons are suppressed, and so a fractionalized spin liquid regime is stable. Similar arguments have also been advanced recently for cases where there are relativistic massless Bose fields [28]. However, these arguments neglect the screening of the instanton-instanton coupling by bound instanton pairs at shorter scales. A number of

other authors [29, 30, 31, 32] have mentioned this screening, and argued that it disrupts the binding of instantons in 2+1 spacetime dimensions; the instantons are always ultimately in a plasma phase, interacting via an exponentially screened interaction, and the spinons carrying dual ‘electric’ charges are consequently permanently confined (these arguments are presented here in Appendix A). Recently, the case for permanent confinement was put on a firmer footing by two of us [33]: an explicit duality mapping of a compact U(1) gauge theory minimally coupled to relativistic fermions showed that the plasma state of instantons was the only stable phase, and so there were no ‘electrically’ charged fermionic spinons. This result implies that the staggered flux and algebraic spin liquids do not exist in two spatial dimensions.

The problem addressed in this paper is the case of a compact U(1) gauge theory coupled to *non-relativistic* matter, with the fermionic spinons forming a Fermi surface. As noted above, this case has been the focus of much attention [2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] in models of the normal state of the cuprates. For this case, Nagaosa [10] has advanced arguments that the dissipation caused by the spinon Fermi surface can suppress the instanton quantum fluctuations, and so stabilize the spin liquid state. However, using the method of Ref. 33, we show below that Nagaosa’s argument also neglects the screening between instantons [34], and that the spinon Fermi surface is generically unstable in two spatial dimensions at zero temperature.

A surviving route to fractionalization in two spatial dimensions arises in physical situations with a Higgs field which does not belong to the fundamental representation of the gauge group [35]. In the spin liquid case, such a Higgs field is provided by the pairing of the spinons [36, 37, 38, 39] in a BCS-like state, and the formation of such a BCS condensate breaks the U(1) gauge symmetry down to  $Z_2$ . The resulting  $Z_2$  gauge theory does permit a deconfined phase, with spinons which carry only  $Z_2$  gauge charges. Note, however, that because of the pairing, there is no spinon Fermi surface in this case either.

For completeness, we also mention the ‘chiral spin liquid’ [37, 40, 41, 42], which allows fractionalization in the presence of time-reversal symmetry breaking, but again without a spinon Fermi surface.

This paper will only be concerned with matter in the fundamental representation of the U(1) gauge group. Our analysis below shows that the only possible exceptions to conventional confinement in such situations arise at special critical points where a certain relevant

coupling in the action accidentally vanishes: at least one parameter has to be tuned to a particular value for this to happen. At such critical points, the gauge theory is not in a conventional deconfined Coulomb phase either, and it is likely that there are no quasiparticle excitations. These points are analogous to the critical Rokhsar-Kivelson points in quantum dimer models [43, 44].

Our analysis of the compact U(1) gauge theory coupled to fermionic spinons on a Fermi surface begins in Section II with a duality mapping to a sine-Gordon-like theory. The renormalization group analysis of the latter theory is presented in Section III and Appendix B. Appendix A contains a discussion of similar physics expressed directly in terms of the statistical mechanics of instantons.

## II. DUALITY MAPPING

A fundamental characteristic of the proposed spin liquid state [4, 5] is the overdamped nature of the transverse gauge field propagator. This damping arises from the low energy fluctuations of the spinon Fermi surface. We denote the components of the compact U(1) gauge field by  $a_\mu$  with  $\mu = 0, x, y$ , the zero'th component representing the imaginary time direction; also by  $a_i$  we denote only the spatial components of  $a_\mu$ . Then, after integrating out the fermionic spinon fields, the action,  $\mathcal{S}$ , for  $a_\mu$  at small wavevectors ( $k$ ) and small frequencies ( $\omega$ ) has the following singular form [4, 5, 6, 9]

$$\mathcal{S} = \int \frac{d^2k d\omega}{8\pi^3} \left[ \frac{1}{2} a_i a_j \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) (|\omega| \sigma(k) + \chi_d k^2) + \frac{\chi}{2} \left( 1 + \frac{\gamma|\omega|}{k} \right) \left( a_0 - \frac{\omega}{k^2} k_i a_i \right)^2 \right]. \quad (1)$$

Here  $\sigma(k)$  is the spinon ‘‘conductivity’’,  $\chi_d$  is the spinon diamagnetic susceptibility,  $\chi$  is the spinon compressibility, and  $\gamma$  is a damping coefficient characteristic of the spinon Fermi surface. All of the terms in  $\mathcal{S}$  are characteristic of a fermionic spinon system with a finite density of states at the Fermi level (for the relativistic case with a Dirac spinon spectrum, there is a vanishing density of states at the Fermi level, and this makes the effective gauge field action quite different [33]). For  $\sigma(k)$ , it is conventional to assume [4, 9]

$$\sigma(k) \sim \begin{cases} \ell & \text{for } k\ell < 1 \\ 1/k & \text{for } k\ell > 1 \end{cases} \quad (2)$$

where  $\ell$  is the mean-free path associated with scattering off static impurities.

Our subsequent analysis is aided by writing  $\mathcal{S}$  in a form which is explicitly gauge invariant. We define the electromagnetic field  $F_{\mu\nu} = p_\mu a_\nu - p_\nu a_\mu$  where  $p_0 = \omega$ ,  $p_i = k_i$ . Then, it is easy to see that

$$\mathcal{S} = \int \frac{d^2k d\omega}{8\pi^3} \left[ \frac{1}{4} \left( \frac{|\omega|\sigma(k)}{k^2} + \chi_d + \frac{1}{e^2} \right) F_{ij}^2 + \frac{1}{2} \left\{ \frac{\delta_{ij}}{e^2} + \frac{\chi k_i k_j}{k^4} \left( 1 + \frac{\gamma|\omega|}{k} \right) \right\} F_{i0} F_{j0} \right]. \quad (3)$$

The expression (3) for  $\mathcal{S}$  also includes a regular  $F_{\mu\nu}^2/(4e^2)$  term in the action for the gauge field; such a term will invariably be generated by integrating out high energy fluctuations of matter fields at short scales.

We may now proceed to dualize the above action for the gauge fields. Before doing so, it is necessary to explicitly account for the compact nature of the U(1) gauge group on the scale of underlying lattice. We do this by discretizing space and time, and writing the compactified lattice version of the action (3) as

$$\begin{aligned} \mathcal{S} = \sum_{x,y,\tau,\tau'} & \left[ \frac{1}{4} (F_{ij}(x, \tau) - 2\pi n_{ij}(x, \tau)) V_t(x - x', \tau - \tau') (F_{ij}(x', \tau') - 2\pi n_{ij}(x', \tau')) \right. \\ & \left. + \frac{1}{2} (\Delta_i (F_{i0}(x, \tau) - 2\pi n_{i0}(x, \tau))) V_l(x - x', \tau - \tau') (\Delta_j (F_{j0}(x', \tau') - 2\pi n_{j0}(x', \tau'))) \right] \\ & + \frac{1}{2e^2} \sum_{x,\tau} (F_{i0}(x, \tau) - 2\pi n_{i0}(x, \tau))^2, \end{aligned} \quad (4)$$

where the transverse and the longitudinal parts of the interaction in the Fourier space read

$$V_t(k, \omega) = \frac{|\omega|\sigma(k)}{k^2} + \chi_d + \frac{1}{e^2}, \quad (5)$$

$$V_l(k, \omega) = \frac{\chi}{k^4} \left( 1 + \gamma \frac{|\omega|}{k} \right). \quad (6)$$

The integers  $n_{\mu\nu}(x, \tau)$  serve to account for the compact nature of the gauge fields, in the spirit of the Villain approximation. On a lattice, the electromagnetic tensor  $F_{\mu\nu} = \epsilon_{\mu\nu\alpha} \epsilon_{\alpha\rho\sigma} \Delta_\rho a_\sigma$ , where  $\Delta_\mu$  is the lattice (discrete) derivative.

Performing the Hubbard-Stratonovich transformation using fields  $c_{\mu\nu}$  residing on the plaquettes of the lattice, we find

$$\begin{aligned} \mathcal{S} = \sum_{x,x',\tau,\tau'} & \left[ c_{ij}(x, \tau) V_t^{-1}(x - x', \tau - \tau') c_{ij}(x', \tau') + \frac{1}{2} e^2 c_{i0}(x, \tau) \left\{ \delta_{ij} \delta_{xx'} \delta_{\tau\tau'} \right. \right. \\ & \left. \left. + \left( \frac{1}{|\Delta|^2} - \left( \frac{1}{e^2} \delta_{xx'} \delta_{\tau\tau'} + V_l(x - x', \tau - \tau') |\Delta|^2 \right)^{-1} \right) \Delta_i \Delta_j \right\} c_{j0}(x', \tau') \right] \\ & + i \sum_{x,\tau} c_{\mu\nu}(x, \tau) (F_{\mu\nu}(x, \tau) - 2\pi n_{\mu\nu}(x, \tau)). \end{aligned} \quad (7)$$

The integral over the gauge fields can now be performed exactly, and it enforces the constraint

$$c_{\mu\nu}(x, \tau) = \frac{1}{2}\epsilon_{\mu\nu\alpha}\Delta_\alpha\Phi(x, \tau). \quad (8)$$

The sum over the integers  $n_{\mu\nu}(x, \tau)$  can also be performed, and it forces the new variable  $\Phi(x, \tau)$  to be an integer as well. From the constraint in Eq. (8) it immediately follows,

$$\Delta_i c_{i0}(x, \tau) = 0, \quad (9)$$

and the corresponding part in Eq. (7), which includes the longitudinal part of the interaction  $V_l$ , completely drops out. Relaxing the integer constraint on  $\Phi$  by introducing a small fugacity  $y$  in the usual way [45], upon return to the continuum limit we find the dual sine-Gordon action for the instantons

$$\begin{aligned} \mathcal{S}_{sG} = \frac{1}{2} \int \frac{d^2k d\omega}{8\pi^3} & \left[ |\Phi(k, \omega)|^2 \left( \frac{\omega^2}{|\omega|\sigma(k)/k^2 + \chi_d + 1/e^2} + e^2 k^2 \right) \right] \\ & - 2y \int d^2x d\tau \cos(2\pi\Phi(x, \tau)). \end{aligned} \quad (10)$$

A corresponding sine-Gordon theory for the relativistic matter case appears in Ref. 28. The problem of confinement in the original theory now reduces to understanding the possible phases of  $\mathcal{S}_{sG}$ , to which we turn in the next section [46].

### III. RENORMALIZATION GROUP ANALYSIS

To be specific, we assume that in the sine-Gordon theory the wavevectors  $|\vec{k}| < \Lambda$ , with  $\Lambda\ell < 1$ , and frequencies  $|\omega| < \Omega$ . To simplify the calculation, we also take  $\Omega \gg (\chi_d + e^{-2})\Lambda^2/\ell$ , so one can write the sine-Gordon field propagator for low momenta and frequencies as

$$G^{-1}(\vec{k}, \omega) = \frac{|\omega|k^2}{\ell} + a_k k^2 + a_\omega \omega^2, \quad (11)$$

where the bare (unrenormalized) coefficients are  $a_k = e^2$ , and  $a_\omega = 0$ . Our methods below are easily generalized to the clean limit case where  $\sigma(k) \sim 1/k$ , and the results are very similar.

Let us integrate out the short-distance modes with  $\Lambda/b < |\vec{k}| < \Lambda$ , and  $|\omega| < \Omega$ , with  $\ln(b) \ll 1$ . The parameters for the remaining long-distance modes are then changed as

$$\ell(b) = \ell, \quad (12)$$

$$a_k(b) = a_k + \frac{1}{2}y^2 e^{-G_>(\vec{x}=0, \tau=0)} \int d^2\vec{x} d\tau |\vec{x}|^2 (e^{G_>(\vec{x}, \tau)} - 1), \quad (13)$$

$$a_\omega(b) = b^2 [a_\omega + y^2 e^{-G_>(\vec{x}=0, \tau=0)} \int d^2\vec{x} d\tau \tau^2 (e^{G_>(\vec{x}, \tau)} - 1)], \quad (14)$$

$$y(b) = b^2 y e^{-\frac{1}{2}G_>(\vec{x}=0, \tau=0)}, \quad (15)$$

where the correlation function is defined as

$$G_>(\vec{x}, \tau) = \int_{\Lambda/b < |\vec{k}| < \Lambda} \frac{d^2\vec{k}}{(2\pi)^2} \int_{-\Omega}^{\Omega} \frac{d\omega}{2\pi} e^{i\vec{k}\cdot\vec{x} + i\omega\tau} G(\vec{k}, \omega). \quad (16)$$

A crucial feature of the above recursion relations is that although initially the coefficient  $a_\omega = 0$ , for  $b > 1$   $a_\omega(b) > 0$ . This expresses a simple physical effect that the interaction between two distant charges in three spacetime dimensions in a medium with a finite polarizability is always Coulombic ( $\sim 1/\sqrt{x^2 + \tau^2}$ ) in three spacetime dimensions [33]. The only apparent way the generation of this term can be avoided would be if the propagator  $G(\vec{k}, \omega)$  would be *completely* independent of frequency, in which case  $G_>(x, \tau) \sim \ln(b)e^{-|x|\Lambda}\delta(\tau)$ , rendering the integral in  $a_\omega(b)$  to be exactly zero. This unphysical situation would correspond to an essentially two dimensional sine-Gordon theory, which would then have the standard Kosterlitz-Thouless transition.

In the present case, on the other hand, we find

$$\frac{d\hat{a}_k}{d\ln(b)} = \frac{\hat{y}^2}{2\pi\hat{a}_k} + O(\hat{y}^3), \quad (17)$$

$$\frac{d\hat{a}_\omega}{d\ln(b)} = 2\hat{a}_\omega + \frac{\hat{y}^2}{4\hat{a}_k} + O(\hat{y}^3), \quad (18)$$

where we have introduced the dimensionless combinations  $\hat{y} = y/\Lambda^2\Omega$ ,  $\hat{a}_k = a_k/\Omega$ , and  $\hat{a}_\omega = a_\omega\Omega/\Lambda^2$ , and introduced a smooth cutoff as described in Appendix B.

As announced earlier, a finite coefficient  $a_\omega$  becomes generated to the second order in fugacity, and then becomes a relevant coupling. With this term included the fugacity flows according to

$$\frac{d\hat{y}}{d\ln(b)} = \left(2 - \frac{1}{8\pi\sqrt{\hat{a}_k\hat{a}_\omega - (1/2\ell)^2}}\right)\hat{y} + O(\hat{y}^3), \quad (19)$$

and always becomes relevant at long length scales. In Eq. (19) we assumed  $\hat{a}_k\hat{a}_\omega \gg (1/2\ell)^2$ ; other limits and more complete expressions are presented in Appendix B. We interpret this

as an instability of the deconfined phase in the original theory, since the instantons are always in the plasma phase, and the interaction between two well-separated instantons is exponentially screened.

Previous renormalization group analyses of instanton effects in U(1) spin liquids by Nagaosa [10], Ioffe and Larkin [26], and Wen [16] accounted *only* for the flow of the instanton fugacity in Eq. (19). Taken on its own, this equation would indeed suggest that there is a regime of parameters ( $8\pi\sqrt{\hat{a}_k\hat{a}_\omega - (1/2\ell)^2} < 1/2$  for Eq. (19)) where the fugacity  $y$  flows to zero, and the instanton effects are negligible: this was the conclusion of these earlier works [10, 16, 26]. However, this conclusion does not hold after accounting for the flows in Eqs. (17,18).

#### IV. CONCLUSIONS

The primary conclusion of this paper is that a two (spatial) dimensional system of fermionic spinons residing on an incipient Fermi surface, and interacting with a compact U(1) gauge force, is generically in a confining phase at zero temperature. Consequently, there are no free spinon-like excitations. When combined with earlier results on the confinement of spinons with a Dirac fermion spectrum [29, 30, 31, 32, 33], our results rule out the existence of a spin liquid phase in two spatial dimensions with deconfined spinons interacting with a U(1) gauge force. In particular, it appears that none of the algebraic, U(1), and SU(2) symmetric spin liquids in Wen's classification [16] are stable phases of matter.

Our analysis also points to the existence of possible (multi)-critical points at which the confinement picture does not directly hold. These correspond to points where accidental cancellations cause the renormalized values of  $\hat{a}_k$  and/or  $\hat{a}_\omega$  to vanish. These are likely quantum critical points between different confining phases, and not conventional Coulomb states. Similar points appeared in analyses of quantum dimer models [43, 44].

Finally, we note that the arguments of this paper do not extend to *three* spatial dimensions. Here, a compact U(1) gauge field can be in a Coulomb phase, and interesting U(1) spin liquids are possible [47].

## Acknowledgments

We are grateful F. Nogueira and A. Sudbo for a number of very constructive and useful discussions. We also thank E. Fradkin, N. Nagaosa and X.-G. Wen for helpful exchanges. This research was supported by NSERC of Canada and the Research Corporation (I.H. and B.S.) and by US NSF Grant DMR-0098226 (S.S.) and DMR-0071611 (G. M.).

## APPENDIX A: SCREENING OF INSTANTON INTERACTIONS

The analysis in the body of the paper, and in the Ref. 33 has been carried out using the dual sine-Gordon representation. While this is technically a useful way to proceed, it does have the disadvantage of obscuring some of the underlying physics. To remedy this, we present our arguments here using the direct partition function for the instantons. For simplicity, we will restrict our discussion here to the case of Dirac fermion spinons, where the ‘relativistic’ nature of the theory does simplify the formalism by making the physics isotropic in spacetime. However, similar arguments can be advanced also for the non-relativistic case. The computations below expand on the remarks in Refs. 29, 30, 31, 32, and were implicitly carried out in Ref. 31. They also connect with some of the arguments presented in Ref. 33, as argued below.

With spinons with a Dirac fermion spectrum, two instantons have an action which depends logarithmically on the separation between them [16, 26, 27]. Let us consider a model of integer ‘charges’  $m_i$  (instantons) at spacetime positions  $\vec{R}_i$ , with the interaction term in the action  $\sum_{i<j} m_i m_j V(\vec{R}_i - \vec{R}_j)$ , where

$$V(\vec{R}) \equiv -K \ln(|\vec{R}|/a), \tag{A1}$$

with  $K$  a coupling constant. We consider such an interaction in  $D$  spacetime dimensions. We are interested here is  $D = 3$ , but the general  $D$  analysis will also allow us to connect with classical results on the Kosterlitz-Thouless transition.

Following the original analysis of Kosterlitz [48], we compute the effective interaction  $V_{\text{eff}}(\vec{R})$  between two well separated test charges: a  $+1$  charge at  $\vec{R}_1$  and a  $-1$  charge at  $\vec{R}_2$ . To lowest order in the instanton fugacity,  $y$ , this interaction is renormalized by instanton dipoles. In the spirit of the renormalization group, we consider only the renormalization by dipoles consisting of opposite charges with spacetime separation between  $a$  and  $a + da$ . So

the dipole consists of a +1 charge at  $\vec{R} + \vec{a}/2$  and a -1 charge at  $\vec{R} - \vec{a}/2$ . As in Ref. 48, to leading order in  $y$  the expression for  $V_{\text{eff}}$  is

$$\exp\left(V_{\text{eff}}(\vec{R}_1 - \vec{R}_2)\right) = \frac{\exp\left(V(\vec{R}_1 - \vec{R}_2)\right)}{Z} \left[ 1 + y^2 \int_a^{a+da} d^D \vec{a} \int d^D \vec{R} \exp\left(-V(\vec{R}_1 - \vec{R} - \vec{a}/2) + V(\vec{R}_1 - \vec{R} + \vec{a}/2) + V(\vec{R}_2 - \vec{R} - \vec{a}/2) - V(\vec{R}_2 - \vec{R} + \vec{a}/2)\right) + \mathcal{O}(y^4) \right] \quad (\text{A2})$$

where  $y$  is the charge fugacity, and the normalization

$$Z = 1 + y^2 \int_a^{a+da} d^D \vec{a} \int d^D \vec{R} + \mathcal{O}(y^4). \quad (\text{A3})$$

The integral in  $Z$  is proportional to the volume of the system: however, this is standard in the low density expansion of any system, and the volume dependence cancels in all physical quantities order-by-order in  $y$ ; the logarithm of the partition function is extensive, and this dependence on the system volume is merely a signal of that. In the above expression for  $V_{\text{eff}}$  it is assumed that any two charges are never less than a distance  $a$  apart, and this cuts off any incipient ultraviolet divergences. Now expand the argument of the exponential in powers of  $a$ . This yields

$$\exp\left(V_{\text{eff}}(\vec{R}_1 - \vec{R}_2)\right) = \frac{\exp\left(V(\vec{R}_1 - \vec{R}_2)\right)}{Z} \left[ 1 + y^2 \int_a^{a+da} d^D \vec{a} \int d^D \vec{R} \exp\left(-K\vec{a} \cdot \left(\frac{\vec{R}_1 - \vec{R}}{(\vec{R}_1 - \vec{R})^2} - \frac{\vec{R}_2 - \vec{R}}{(\vec{R}_2 - \vec{R})^2}\right) + \mathcal{O}(a^3)\right) + \mathcal{O}(y^4) \right] \quad (\text{A4})$$

The integral over  $\vec{a}$  can be performed, and to leading order in  $y$  and  $a$  we obtain

$$V_{\text{eff}}(\vec{R}_1 - \vec{R}_2) = -K \ln(|\vec{R}_1 - \vec{R}_2|/a) + y^2 \frac{S_D K^2 a^{D+1} da}{2D} \int d^D \vec{R} \left( \frac{\vec{R}_1 - \vec{R}}{(\vec{R}_1 - \vec{R})^2} - \frac{\vec{R}_2 - \vec{R}}{(\vec{R}_2 - \vec{R})^2} \right)^2 \quad (\text{A5})$$

$$= -K \ln(|\vec{R}_1 - \vec{R}_2|/a) + y^2 \frac{S_D K^2 a^{D+1} da}{2D} \int d^D \vec{R} \frac{(\vec{R}_1 - \vec{R}_2)^2}{(\vec{R}_1 - \vec{R})^2 (\vec{R}_2 - \vec{R})^2} \quad (\text{A6})$$

where  $S_D$  is the surface area of a sphere in  $D$  dimensions. Note that the volume dependence has cancelled as expected. The above is, of course, an adaptation of the computation performed by Kosterlitz [48].

Now, notice that there is a crucial distinction in the integral in (A6) between  $D = 2$  and  $D = 3$ . In  $D = 2$ , the integral *reproduces* the functional form of the bare logarithmic

interaction, and so the renormalization of the potential can indeed be accounted for by a dielectric constant:

$$V_{\text{eff}}(\vec{R}_1 - \vec{R}_2) = -K \ln(|\vec{R}_1 - \vec{R}_2|/a) + y^2 \frac{S_2^2 K^2 a^3 da}{2} \left[ \ln(|\vec{R}_1 - \vec{R}_2|/a) + \text{constant} \right] \quad , \quad D = 2, \quad (\text{A7})$$

where the ultraviolet cutoff radius around each charge is used to make the integral finite. This leads to the usual Kosterlitz recursion relations [48] in  $D = 2$ .

Turning, finally, to  $D = 3$ , the integral over  $\vec{R}$  in (A6) now yields

$$V_{\text{eff}}(\vec{R}_1 - \vec{R}_2) = -K \ln(|\vec{R}_1 - \vec{R}_2|/a) + y^2 \frac{S_3^2 \pi^2 K^2 a^4 da}{24} |\vec{R}_1 - \vec{R}_2| \quad , \quad D = 3. \quad (\text{A8})$$

There was now no need to apply any ultraviolet cutoff. Note that the bare logarithmic interaction is *not reproduced* under renormalization—this is the central point. Instead, we have generated a *linear interaction*, which has a much stronger dependence on the separation between the charges. It is also crucial to note the sign of this linear interaction: it is such as to weaken the interaction between oppositely charged particles, and at large enough  $|\vec{R}_1 - \vec{R}_2|$  the effective interaction between these opposite charges actually becomes repulsive. Of course, we cannot trust the renormalization group beyond these scales. Nevertheless, the lesson is quite clear: the effect of the dipole is to substantially weaken the interaction between the charges, suggesting a ubiquitous instability to a plasma phase.

Continuing this line of argument, we can also look at the screening of the generated linear interaction. Using exactly the same derivation as above, the integral in the second term on the right hand side of (A5) will be replaced by

$$\int d^3 \vec{R} \left( \frac{\vec{R}_1 - \vec{R}}{|\vec{R}_1 - \vec{R}|} - \frac{\vec{R}_2 - \vec{R}}{|\vec{R}_2 - \vec{R}|} \right)^2 \quad (\text{A9})$$

This integral is now divergent at large  $\vec{R}$ , which reconfirms our conclusion that the screening effect of the largest scales  $\vec{R}$  is extremely disruptive.

The Eq. (A8) may also be understood as the expansion of the screened interaction found in Ref. 33 in small fugacity. Considering the electrostatics of logarithmically interacting charges bound in dipole pairs, the effective interaction due to an external charge, in the Fourier space, becomes,

$$V_{\text{eff}}(q) = \frac{K}{|q|^3 + \chi q^2}, \quad (\text{A10})$$

in  $D = 3$ , where  $\chi \sim y^2$  is the finite polarizability of the medium [33]. At a large distance  $R$  this translates into  $V_{eff}(R) \sim 1/(4\pi\chi R)$ . Expanding in small  $\chi$ , on the other hand, yields

$$V_{\text{eff}}(q) = \frac{K}{|q|^3} - \frac{K\chi}{q^4} + O(\chi^2). \quad (\text{A11})$$

The first term in the expansion then represents the bare logarithmic interaction in Eq. (A8), while the second one may be identified with the next, linear correction.

## APPENDIX B: COMPUTATION FOR RENORMALIZATION ANALYSIS

Here we discuss the smooth cutoff procedure we devised to compute  $G_{>}(x, \tau)$  in Eq. (16), and to arrive at the flow equations (17-19). With  $a_\omega = 0$  in the unrenormalized theory,

$$G_{>}(x, \tau) = \frac{1}{2\pi e^2} \int_{\Lambda/b}^{\Lambda} \frac{dk}{k} J_0(kx) \int_{-\Omega}^{\Omega} \frac{d\omega}{2\pi} \frac{e^{i\omega\tau}}{1 + |\omega|/(e^2\ell)}. \quad (\text{B1})$$

First, we write

$$\int_{\Lambda/b}^{\Lambda} \frac{dk}{k} J_0(kx) = \int_{\Lambda/b}^{\infty} \frac{dk}{k} J_0(kx) - \int_{\Lambda}^{\infty} \frac{dk}{k} J_0(kx) \quad (\text{B2})$$

and then introduce a smooth cutoff [33] as

$$\int_{\Lambda}^{\infty} \frac{dk}{k} J_0(kx) \rightarrow \int_0^{\infty} \frac{dk}{k} J_0(kx) \frac{k^2}{k^2 + \Lambda^2}. \quad (\text{B3})$$

This leads to

$$\frac{1}{2\pi e^2} \int_{\Lambda/b}^{\Lambda} \frac{dk}{k} J_0(kx) \rightarrow -\frac{x\Lambda \ln(b)}{2\pi e^2} \left. \frac{dK_0(z)}{dz} \right|_{z=x\Lambda}, \quad (\text{B4})$$

with  $K_0(z)$  as the Bessel function. Similarly, for  $\Omega/e^2\ell \ll 1$  we may neglect the frequency dependence and approximate the second integral in Eq. (B1) as

$$\int_{-\Omega}^{\Omega} \frac{d\omega}{2\pi} e^{i\omega\tau} \rightarrow \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega\tau} e^{-\pi(\omega/(2\Omega))^2}, \quad (\text{B5})$$

where we have introduced the Gaussian instead of the sharp cutoff in the frequency space.

This gives

$$\int_{-\Omega}^{\Omega} \frac{d\omega}{2\pi} \frac{e^{i\omega\tau}}{1 + |\omega|/(e^2\ell)} \rightarrow \frac{\Omega}{\pi} e^{-(\tau\Omega)^2/\pi}. \quad (\text{B6})$$

Inserting the last two expressions into Eqs. (13) and (14) yields the  $\sim y^2$  terms in the recursion relations Eqs. (17) and (18).

Assuming  $a_\omega > 0$ , the frequency cutoff  $\Omega$  may be taken to infinity in the computation of  $G_{>}(0, 0)$  appearing in the Eq. (15). This leads to

$$\frac{d\hat{y}}{d\ln(b)} = \left[ 2 - \frac{\ell}{4\pi D^{1/2}} \left( 1 - \frac{2}{\pi} \arctan \left( \frac{1}{D^{1/2}} \right) \right) \right] \hat{y} + O(\hat{y}^3), \quad (\text{B7})$$

for  $D > 0$ , where

$$D = 4\hat{a}_k\hat{a}_\omega\ell^2 - 1. \quad (\text{B8})$$

For  $D < 0$ , one finds

$$\frac{d\hat{y}}{d\ln(b)} = \left[ 2 - \frac{\ell}{4\pi^2|D|^{1/2}} \ln \left( \frac{1 + |D|^{1/2}}{1 - |D|^{1/2}} \right) \right] \hat{y} + O(\hat{y}^3). \quad (\text{B9})$$

For completeness, let us also mention the case  $a_\omega(b) \equiv 0$ , which would arise if one would neglect  $\sim y^2$  terms in the recursion relations. One then needs to keep a finite cutoff  $\Omega$ , to find

$$\frac{d\hat{y}}{d\ln(b)} = \left[ 2 - \frac{\ell}{4\pi^2} \ln \left( 1 + \frac{1}{\ell\hat{a}_k} \right) \right] \hat{y} + O(\hat{y}^3). \quad (\text{B10})$$

Because to the lowest order in  $y$  neither  $\ell$  nor  $\hat{a}_k$  scale, the last equation would suggest that fugacity is irrelevant for small enough  $\hat{a}_k$ , for example. This conclusion, however, inevitably breaks down to the next order in  $y$ , as discussed in the body of the paper.

Note also the similarity between the Eq. (B10) and Eq. (11) in Ref. 10, upon identification of  $\ell$  with  $\gamma$ , and  $a_k$  with  $g^2$ . The main difference is that in Nagaosa's renormalization scheme the coupling  $g^2$  (analogous to our coefficient  $a_\omega$ ) appears *irrelevant* by naive power counting, which then suggest that fugacity should become irrelevant as well. This conclusion, however, changes to the next order in fugacity, in which the fugacity always becomes relevant even within Nagaosa's renormalization scheme, and much like in our calculation.

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