

# Tunneling gap of laterally separated quantum Hall systems

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We use a method of matched asymptotics to determine the energy gap of two counter-propagating, strongly interacting, quantum Hall edge states. The microscopic edge state dispersion and Coulomb interactions are used to precisely constrain the short-distance behavior of an integrable field theory, which then determines the low energy spectrum.

For many solid-state materials, modern density-functional theory allows accurate computation of electronic properties *ab initio*. This success has not been extended to correlated electron systems like transition metal oxides, including the cuprate superconductors, and low-dimensional semiconductor heterostructures. In such systems, the reduced kinetic energy bandwidth enhances the importance of the correlations, and in some cases this leads to new quantum phases of matter. Quantum field-theoretic methods can often lead to a good understanding of the correlated phases, but their quantitative predictive power is limited: different energy scales can be related to each other, but at least one has to be measured experimentally. In this paper we shall describe one strongly correlated system in which it is possible to make a precise quantitative connection between the microscopic Hamiltonian and the electronic spectrum.

We consider the recent experiments of Kang *et al.* [1] which studied tunneling between the counter-propagating edge states of two laterally separated quantum Hall states in two-dimensional electron gases (2DEG) in GaAs quantum wells (see Fig. 1). The 2DEGs are separated by a smooth barrier so the electrons in the edge states move ballistically along the barrier, while mixing via a weak matrix element for tunneling under the barrier, and strongly interacting via the Coulomb interaction. These coupled edge states constitute the correlated electron system of interest here. As has been argued by Kang *et al.* [1], and more completely by Mitra and Girvin [2], the magnetic field and bias voltage dependence of the tunneling conductance act as sensitive probes of this system. In particular, the field interval over which a zero-bias peak is present determines the energy gap above the quantum ground state.

This paper will make a microscopic computation of the energy gap of the coupled edge state system. We use the method of matched asymptotics: we combine the results of two separate calculations, valid on different length scales, over their intermediate regime of overlapping validity. It is the weak bare tunneling matrix element which leads to a clear-cut separation of scales crucial to our approach. At short scales, we will use a perturbative, microscopic calculation to account for the

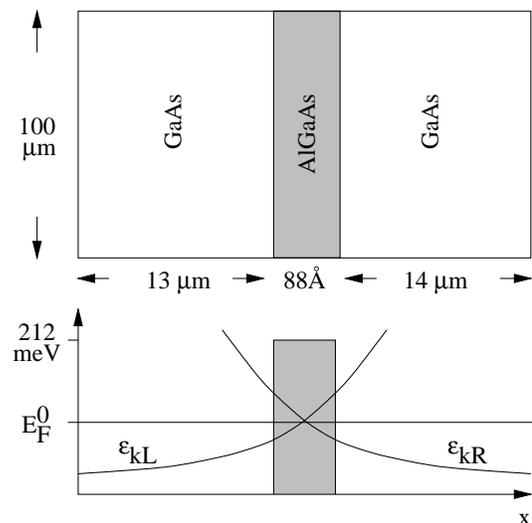


FIG. 1. The experiment of Kang *et al.* [1]: two 2DEGs are laterally separated by an atomically precise tunneling barrier, with a height of 212 meV [2]. The differential conductance is measured across the junction, with peaks occurring at zero bias for filling factors  $\nu \approx 1.2$ -1.5 and  $\nu \approx 2.2$ -2.5. The edge state dispersions are sketched for a barrier of infinite height; a gap appears for non-zero tunneling.

Coulomb interactions between the edge states (in the spirit of density-functional theory). At larger scales, we will use an integrable quantum field theory to determine the energy gap induced by the tunneling. However, the results of the quantum field theory depend sensitively on the nature of the cutoff regulating its *short* distance behavior: we will show how this is precisely specified by the *long* distance limit of the first calculation. The same quantum field theory has also been used in other recent theoretical works [2,3]; however, these works have used an ad-hoc specification of its cutoff. Our results show that there is no ambiguity in regulating the field theory, and that there is a precise matching procedure which connects it to the microscopic band structure.

We consider the Hamiltonian  $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_t$ . For an infinitely high barrier the one-dimensional motion of the electrons on the two edge states ( $\sigma = L, R$ ) is represented by [4-6]

$$\hat{H}_0 = \sum_k \left( \epsilon_{kL} \hat{c}_{kL}^\dagger \hat{c}_{kL} + \epsilon_{kR} \hat{c}_{kR}^\dagger \hat{c}_{kR} \right), \quad (1)$$

where  $\hat{c}_{k\sigma}^\dagger$  is a creation operator for a fermion in an orbital with one-dimensional momentum,  $k$ , parallel to the barrier. The dispersion  $\epsilon_{k\sigma}$  was determined by a solution of the Schrödinger equation in the lowest Landau level, assuming complete spin polarization; we will need the full functional form of this dispersion, and not just a linearized Fermi velocity near the Fermi level. For zero bias voltage the dispersion satisfies  $\epsilon_{kL} = \epsilon_{-kR} = \hbar\omega_c \nu_a(k)$ , where  $\omega_c$  is the cyclotron frequency. With  $\ell$  the magnetic length, and  $2a$  the width of the barrier,  $\nu_a(k)$  depends on  $k$  and  $a$  only through  $\nu_a(k) = w(k\ell + a/\ell)$ . The single-particle wavefunctions in the Landau gauge are  $\psi_{k\sigma}(x, y) = \phi_{k\sigma}(x) e^{iky}/\sqrt{L}$ , where  $\phi_{k\sigma}(x)$  vanishes at the barrier;  $L$  is the (effectively infinite) length of the barrier.

The Coulomb interaction among the electrons in the edge states is

$$\hat{H}_{\text{int}} = \frac{1}{2L} \sum_{kk'q\sigma\sigma'} V_{kk'q}^{\sigma\sigma'} \hat{c}_{k+q\sigma}^\dagger \hat{c}_{k'-q\sigma'}^\dagger \hat{c}_{k'\sigma'} \hat{c}_{k\sigma}, \quad (2)$$

where the matrix elements

$$V_{kk'q}^{\sigma\sigma'} = \int dx_1 \int dx_2 \phi_{k+q\sigma}^*(x_1) \phi_{k\sigma}(x_1) V_C(q, x_1 - x_2) \phi_{k'-q\sigma'}^*(x_2) \phi_{k'\sigma'}(x_2) \quad (3)$$

are computed in terms of the wavefunctions,  $\phi_{k\sigma}(x)$ , of the edge states in the direction transverse to the barrier, with

$$V_C(q, x) = \frac{e^2}{\epsilon} \int dk \frac{e^{iqx}}{\sqrt{k^2 + q^2} + q_{\text{TF}}}. \quad (4)$$

Here  $\epsilon$  is the dielectric constant of GaAs ( $\epsilon = 12.6$ ) and  $q_{\text{TF}}$  is the Thomas-Fermi wavevector accounting for screening among the edge-state electrons. Finally, we write the tunneling between the edge states as

$$\hat{H}_t = -t \int dx \hat{O}(x), \quad (5)$$

$$\hat{O}(x) = \frac{1}{L} \sum_{kk'} \hat{c}_{kL}^\dagger \hat{c}_{k'R} e^{i(k-k')x} + \text{h.c.} \quad (6)$$

In the absence of interactions this operator causes an energy gap of  $\Delta_0 = 2t$  in the spectrum of  $\hat{H}_0 + \hat{H}_t$ .

As  $t$  is very small compared to the other energy scales, it is useful to first consider the physics for  $t = 0$ . The system  $\hat{H}_0 + \hat{H}_{\text{int}}$  is a canonical example of a gapless Tomonaga-Luttinger (TL) liquid. One of its characterizations is the power-law correlator

$$\langle \hat{O}(x) \hat{O}(0) \rangle = \frac{2A(\beta^2)^2 \Lambda^{2-4\beta^2}}{|x|^{4\beta^2}} \text{ for } x \rightarrow \infty, t = 0. \quad (7)$$

Here  $\beta^2$  is a TL liquid exponent, and  $A(\beta^2)$  is a dimensionless function; both quantities depend upon the complete microscopic details of  $\hat{H}_0 + \hat{H}_{\text{int}}$ . The inverse length scale,  $\Lambda$ , can be chosen at our convenience: only the combination  $A(\beta^2)\Lambda^{1-2\beta^2}$  is determined by (7), and any change in our choice for  $\Lambda$  will lead to a corresponding change in  $A(\beta^2)$ . None of our results will depend upon this choice. For the free particle case ( $V_C = 0$ ), a simple computation shows that  $\beta^2 = 1/2$  and  $A(1/2) = 1/(2\pi)$ , independent of  $\Lambda$ . For  $V_C \neq 0$ , we computed  $\beta$  and  $A(\beta^2)$  both by perturbation theory in  $V_C$  (valid for  $\beta^2$  near  $1/2$ ). This was the time-consuming part of our calculation, and in principle, there is no problem in extending this computations to higher orders. For now, let us regard  $\beta$  and  $A(\beta^2)$  as known quantities (further details of their computation appear below) and proceed to a discussion of the consequences of  $\hat{H}_t$ .

The effects of  $\hat{H}_t$  are more easily discussed in a bosonized theory of the TL liquid [7]. This is expressed in terms of the continuum, dimensionless, scalar field  $\hat{\varphi}(x)$  with the Hamiltonian ( $\hbar = 1$ )

$$\hat{H}_{\text{TL}} = \frac{v_F}{2} \int dx [(\partial_x \hat{\varphi})^2 + \hat{\pi}^2], \quad (8)$$

where  $\hat{\pi}$  is the canonically conjugate momentum to  $\hat{\varphi}$ , and  $v_F$  is the renormalized Fermi velocity. The standard bosonization mapping [7] also shows that the operator  $\hat{O}(x)$  is proportional to  $\cos(\sqrt{8\pi}\beta\hat{\varphi}(x))$ , and so we conclude that the long-distance properties of the fermionic theory  $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_t$  are described by the sine-Gordon field theory with Hamiltonian

$$\hat{H}_{\text{SG}} = \hat{H}_{\text{TL}} - 2\alpha \int dx \cos(\sqrt{8\pi}\beta\hat{\varphi}(x)). \quad (9)$$

This is a renormalizable and integrable quantum field theory, and all its properties are (in principle) determined in terms of  $\beta$  and  $\alpha$  after suitable renormalization (normal-ordering) of the cosine interaction. Following the convention of Zamolodchikov [8], we normalize the cosine interactions by

$$\langle \cos(\sqrt{8\pi}\beta\hat{\varphi}(x)) \cos(\sqrt{8\pi}\beta\hat{\varphi}(0)) \rangle_{\text{SG}} = \frac{1}{2|x|^{4\beta^2}}, \quad (10)$$

for either  $\alpha \rightarrow 0$  and all  $x$ , or in the short-distance limit,  $x \rightarrow 0$  and all  $\alpha$  [9]. The theory (9) contains a soliton and an antisoliton with mass  $M$ , as well as bound states (“breathers”) with masses  $m_n = 2M \sin(\pi n \xi/2)$ ,  $n = 1, 2, \dots, < 1/\xi$ , where  $\xi = \beta^2/(1 - \beta^2)$ , and  $\alpha$  and  $M$  are related by [8]

$$\frac{\alpha}{v_F} = \frac{\Gamma(\beta^2)}{\pi\Gamma(1 - \beta^2)} \left[ \frac{M}{v_F} \frac{\sqrt{\pi} \Gamma(\frac{1+\xi}{2})}{2\Gamma(\frac{\xi}{2})} \right]^{2-2\beta^2}. \quad (11)$$

Before we can apply (11) we need to determine  $\alpha$ . It is tempting to do this by using the standard representation

of the fermionic fields in terms of exponentials of  $\hat{\varphi}$  and  $\hat{\pi}$  in the expression for  $\hat{O}(x)$ , and thus map  $\hat{H}_t$  to the cosine term in  $\hat{H}_{\text{sG}}$ . However, this is incorrect: this mapping implicitly assumes that the fermions are free ( $V_C = 0$ ), and does not properly account for the essential, non-universal renormalization of the overall scale of  $\hat{O}(x)$ . Instead, the proper answer is obtained with the realization that the results (7) and (10) in fact refer to the same regime of  $x$ : the long-distance limit of  $\hat{H}_0 + \hat{H}_{\text{int}}$  is described by the TL liquid behavior in (7), and this co-incides with the short-distance limit of the continuum theory  $\hat{H}_{\text{sG}}$  as described by the TL liquid behavior in (10). In this manner we may deduce the proportionality constant between  $\hat{O}(x)$  and  $\cos(\sqrt{8\pi}\beta\hat{\varphi}(x))$ , and so obtain

$$\alpha = tA(\beta^2)\Lambda^{1-2\beta^2}. \quad (12)$$

With the knowledge of  $\beta$  and  $A(\beta^2)$  the energy gap,  $\Delta$ , can be obtained from (11)–(12), via

$$\Delta = 2 \min(m_1, M). \quad (13)$$

In particular  $\Delta = \Delta_0$  for the non-interacting case,  $\beta^2 = 1/2$ , as expected.

Before we discuss our final numerical results for  $\Delta$ , let us present a few details of the promised computation of  $\beta$  and  $A(\beta^2)$ . An expansion of the TL correlator (7) for  $\beta^2$  near  $1/2$  yields

$$\langle \hat{O}(x)\hat{O}(0) \rangle = \frac{1}{2\pi^2 x^2} \times [1 + (\beta^2 - \frac{1}{2})(4\pi A'(\frac{1}{2}) - 4 \ln(x\Lambda)) + \dots]. \quad (14)$$

On the other hand we calculate the correlation function of the tunneling operator in the fermionic theory with  $t = 0$  to first order in  $\hat{H}_{\text{int}}$ ,

$$\langle \hat{O}(x)\hat{O}(0) \rangle = \frac{1}{2\pi^2 x^2} \left[ 1 + \frac{e^2/(\epsilon\ell)}{\hbar\omega_c} \frac{g(x/\ell)}{\pi} + \dots \right], \quad (15)$$

and extract the asymptotic behavior of  $g(x)$ ,

$$g(x) = c_1 \ln x + c_2 + O(x^{-2}) \text{ for } x \rightarrow \infty. \quad (16)$$

Comparison with (14) leads to expressions for  $\beta^2$  and  $A'(\frac{1}{2})$  in terms of  $c_1$  and  $c_2$ . It remains to determine  $c_1$  and  $c_2$  from  $g(x)$ . It is straightforward to calculate the required correlation function in terms of a two-particle Green function

$$\langle \hat{O}(x)\hat{O}(0) \rangle = \sum_{kk'q\sigma\sigma'} e^{iqx} \langle \text{T} \hat{c}_{k'\sigma'}(\tau_2) \hat{c}_{k\bar{\sigma}}(\tau_3) \hat{c}_{k+q\sigma}^\dagger(\tau_1) \hat{c}_{k'-q\sigma'}^\dagger(\tau_4) \rangle, \quad (17)$$

for vanishing imaginary times, with  $\tau_1 > \tau_2 > \tau_3 > \tau_4$ . Up to first order in  $V_C$  we obtain

$$\frac{g(x)}{x^2} = \int_0^\infty dq \int_{-q}^0 dk \int_0^q dk' F(k, k', q) \cos(k + q - k')x, \quad (18)$$

where  $F$  and its arguments are dimensionless,

$$\frac{e^2/(\epsilon\ell)}{\hbar\omega_c} F(k\ell, k'\ell, q\ell) = \frac{V_{kk'q}^{\text{LR}}}{\epsilon_{k+qL} - \epsilon_{kL} + \epsilon_{k'-qR} - \epsilon_{kR}}. \quad (19)$$

It is difficult to determine the logarithmic asymptotics numerically from this integral over an oscillatory function. We therefore consider the Mellin transform  $G(s)$  of  $g(x)$ , which is defined by

$$G(s) = \int_1^\infty dx x^{s-1} g(x), \quad s < 0. \quad (20)$$

The following particular transform is important for our purposes,

$$\int_1^\infty dx x^{s-1} \frac{(\ln x)^n}{x^m} = \frac{(-1)^{n+1} n!}{(s-m)^{n+1}}, \quad (21)$$

which shows that powers of logarithms transform into poles in the Laurent expansion of  $G(s)$ . In the present case we expect that

$$G(s) = \frac{c_1}{s^2} - \frac{c_2}{s} + O(s^0) \text{ for } s \rightarrow 0. \quad (22)$$

Performing two partial integration w.r.t  $q$  in (18) leads to the function  $\tilde{g}(p)$ , a Fourier-like representation of  $g(x)$ ,

$$g(x) = \int_0^\infty dp (1 - \cos px) \frac{\partial^2 \tilde{g}(p)}{\partial p^2}, \quad (23)$$

$$\tilde{g}(p) = \int_0^\infty dk \int_0^\infty dk' \sum_{\lambda=\pm 1} F(-\lambda k, \lambda k', \lambda(k+k'+p)), \quad (24)$$

on which the integration in (20) can be performed, with the result (primes mean derivatives w.r.t.  $p$ )

$$G(s) = \int_0^\infty dp \left[ -\frac{\Gamma(s) \cos \frac{\pi s}{2}}{p^s} + \frac{{}_1F_2(\frac{s}{2}, 1 + \frac{s}{2}, \frac{1}{2}, -\frac{p^2}{4}) - 1}{s} \right] \tilde{g}''(p) \quad (25)$$

$$= \frac{\gamma s - 1}{s} \int_0^\infty dp p^{-s} \tilde{g}''(p) + O(s^0), \quad (26)$$

where  $\gamma \approx 0.577$  is Euler's constant. For  $p \rightarrow 0$  it can be shown that  $\tilde{g}''(p) \propto V_{000}^{\text{LR}}/v_{\text{F}}^0 p$ ; a partial integration then yields

$$G(s) = \frac{\gamma s - 1}{s^2} \int_0^\infty dp (1 - s \ln p) (p \tilde{g}''(p))' + O(s^0), \quad (27)$$

from which we obtain, by comparison with (22), and some rearrangement,

$$c_1 = \lim_{p \rightarrow 0} p \tilde{g}''(p) = \frac{\omega_c \ell}{v_{\text{F}}^0} \frac{V_{000}^{\text{LR}}}{e^2/(\epsilon\ell)}, \quad (28)$$

$$\frac{c_2}{c_1} = \int_0^\infty dp \left[ \frac{\tilde{g}''(p)}{c_1} - \frac{e^{-p}}{p} \right], \quad (29)$$

where an integral representation for  $\gamma$  was used to subtract the pole at the origin. The integral in (29) can be performed, with the result

$$\frac{c_2}{c_1} = \lim_{p \rightarrow 0} \int_0^\infty dk \int_0^\infty dk' \frac{\partial}{\partial p} \left[ \frac{e^{-(k+k'+p)}}{k+k'+p} - \sum_{\lambda=\pm 1} \frac{F(-\lambda k, \lambda k', \lambda(k+k'+p))}{c_1} \right], \quad (30)$$

which we evaluate numerically. To obtain the matrix elements (3) we calculated the integral in (4), which is more easily evaluated in terms of a Bessel function, ( $a, b > 0$ )

$$\int_0^\infty \frac{dp \cos p}{\sqrt{p^2 + a^2 + b}} = \int_0^\infty dx \frac{ax e^{-bx}}{\sqrt{1+x^2}} K_1(a\sqrt{1+x^2}), \quad (31)$$

and obtained the real-space wavefunctions of the lowest Landau level edge states from the solution of the Schrödinger equation. This completes the calculation of  $\beta$  and  $A'(1/2)$ .

Finally, the knowledge of  $c_1$  and  $c_2$  allows determination of the energy gap. Eliminating  $\alpha$  from (11)–(12), and expanding in  $\beta^2 - \frac{1}{2}$ , we obtain

$$\frac{M}{t} = 1 + 2(\beta^2 - \frac{1}{2}) \left[ \pi A'(\frac{1}{2}) + \ln \frac{e^\gamma t}{2v_F \Lambda} \right] + \dots \quad (32)$$

To this order, it is permissible to use the bare Fermi velocity  $v_F^0 = \omega_c \ell w'(a/\ell)$ . Matching the expansions in (14) and (16) to express the results in terms of  $c_{1,2}$ , the final result for the energy gap in the presence of interactions,  $\Delta = 2M$ , is

$$\frac{\Delta}{\Delta_0} = 1 + \frac{c_1}{2\pi} \frac{e^2/(\epsilon\ell)}{\hbar\omega_c} \left[ \ln \frac{4\hbar v_F^0}{\Delta_0 \ell} + \frac{c_2}{c_1} - \gamma \right], \quad (33)$$

where the microscopic scale  $\Lambda$  has cancelled. Note that only microscopic parameters appear in this equation; except for the tunneling  $t$  itself (which enters via the bare gap,  $\Delta_0 = 2t$ ), they all depend only on properties of the two 2DEGs without tunneling. It is also obvious that  $t$  enters into  $\Delta$  in a non-perturbative fashion.

Our results for  $\Delta$  are shown in Table I. We find that the correlated gap is larger by a factor of 2-3 than the bare gap. Compared with the Hartree-Fock results of Mitra and Girvin [2] our results for  $\Delta$  are larger by about 10 % for the experimental barrier width of 88 Å and by about 50 % for a smaller barrier of 52 Å. We conclude that the gap is significantly enhanced by Coulomb correlations especially for strong tunneling.

The gap can be related to the filling factor range  $\nu = \nu^* \pm \delta\nu$  where a zero-bias peak is observed in the differential conductance [1,2]; this signal is expected when the Fermi energy falls inside the gap. From our results its appearance is predicted in a range of  $\delta\nu \approx \Delta/(\hbar\omega_c) \approx 0.01$ , much smaller than the experimentally

$2a/l$	$2a$ [Å]	$v_F^0/(\omega_c \ell)$	$\Delta_0$ [K]	$q_{TF}\ell$	$2\beta^2$	$\Delta$ [K]
0.84	88	1.47	0.52	0.05	0.64	1.49
0.84	88	1.47	0.52	0.10	0.76	1.19
0.50	52	1.33	4.00	0.05	0.58	9.25
0.50	52	1.33	4.00	0.10	0.72	7.83

TABLE I. Numerical results for the correlated tunneling gap  $\Delta$  [Eq. (33)] and the TL liquid exponent  $2\beta^2$ , obtained for a magnetic field of  $B = 6$  T, corresponding to magnetic length  $\ell = 105$  Å and cyclotron energy  $\omega_c = 10.4$  meV (based on the effective mass  $m^* = 0.067m$  for GaAs), and several values of the barrier width and Thomas-Fermi screening vector  $q_{TF}$ . The barrier width determines the tunneling gap  $\Delta_0$  in the absence of interactions and was calculated in Ref. [2].

observed  $\delta\nu \approx 0.15$ . At present the reason for this difference is unclear. It may be due to disorder in the sample, which has been suggested to further increase the gap [2]. Another puzzle is that for the non-interacting system the two branches of the dispersion are predicted to cross above the second Landau level, so that the zero-bias conductance peak is expected near  $\nu^* \approx 2$ , but is found experimentally at  $\nu^* \approx 1$  (our approach does not provide an estimate of  $\nu^*$ ). In any case it may be worthwhile to repeat the experiment with a narrower barrier, since in this case the larger bare tunneling amplitudes and Coulomb matrix elements should make correlations dominate over possible disorder effects.

In conclusion, we have provided a microscopic and quantitative computation of an observable in a correlated electron system, the energy gap for lateral tunneling between two interacting quantum Hall systems, and compared our result to experiment. Our method of matched asymptotics should find applications in other correlated systems.

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