PREFACE

The following review article was published in a conference proceeding in 1992. I add the following remarks to place it in the context of recent developments:

- With the modern consensus on the meaning of spin-charge separation [1,2], spin and charge are separated in a BCS superconductor. So the title of the paper should more properly read *"Stable hc/e vortices in a superconductor near a normal state with spin-charge separation"*. This change in terminology does not modify the discussion of the physical properties of the various phases.

- The mean-field theory presented here ignores the compactness of the $U(1)$ gauge field $\vec{a}$, and the related effects of instanton fluctuations. Such effects will not modify the structure of the phases, but will make the total $\vec{a}$-flux in vortices uncertain modulo $2\pi$ [3,4]. From the first equation in (17), this implies that only the value $n_\Delta (\text{mod } 2)$ is physically significant.

- The "vison" excitation of Senthil and Fisher [5] is the vortex in Normal State II with with $n_\Delta (\text{mod } 2) = 1$; the $b$ boson is not condensed in this phase, and so there is no significance to the value of $n_b$. In the superconductor, finiteness of the free energy $F$ is imposed by (17,18), and this requires that the vison must trap a flux of an odd multiple of $hc/2e$.

- The free energy $F$ displays the same "flux regeneration effect" in a cylindrical geometry under the protocol described in Ref 6, and represents a simple way of understanding the effect. Indeed, this effect requires the enhanced stability of $hc/e$ vortices discussed here, for otherwise $hc/2e$ vortices generated by thermal or quantum fluctuations as one moves into the normal state will annihilate the flux in the center of the cylinder. So a direct search for $hc/e$ vortices, proposed in [7] and reviewed in the following paper, represents an alternative experimental route.

I thank T. Senthil for many useful discussions. Some related comments have been made recently in [8]

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# STABLE $hc/e$ VORTICES IN SUPERCONDUCTORS WITH SPIN-CHARGE SEPARATION

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A phenomenological model, $F$, of the superconducting phase of systems with spin-charge separation and antiferromagnetically induced pairing is studied. Above $H_{c1}$, magnetic flux can always pierce the superconductor in vortices with flux $hc/2e$, but regimes are found in which vortices with flux $hc/e$ are preferred. Little-Park and other experiments, which examine periodicities with a *varying* magnetic field, always observe a period of $hc/2e$. The low energy properties of a symplectic large-$N$ expansion of a model of the cuprate superconductors are argued to be well described by $F$. This analysis and some normal state properties of the cuprates suggest that $hc/e$ vortices should be stable at the lowest dopings away from the insulating state at which superconductivity first occurs.

## 1. INTRODUCTION

Much attention has recently focussed on the anomalous normal state properties of the cuprate superconductors[1,2]. A promising description of the high temperature state of these materials has emerged from recent gauge theories[3]. These theories[2,3,4] *assume* that due to strong correlations in the $CuO_2$ layers, the physics at intermediate length scales is best described by a separation of the spin and charge degrees of freedom of the underlying holes. This separation can be encapsulated by the decomposition of the creation operator $d^\dagger_{i\alpha}$ for holes on the $Cu$ $d$-orbital into the following

$$d^\dagger_{i\alpha} = f^\dagger_{i\alpha} b_i \tag{1}$$

where $i$ is a site label, $\alpha = \uparrow, \downarrow$ is the spin index, $f$ is a fermion annihilation operator, and $b$ a boson annihilation operator. The degrees of freedom of the hole have separated into a fermionic spinon, $f_\alpha$ which carries spin but no charge, and a bosonic holon which carries charge but no spin. (Theories with a partial separation of spin and charge are also possible but will be ignored here for simplicity; see Ref 5.) It is now assumed that there exists an intermediate length scale at which the system is well described by the independent propagation of the $b$ and $f^\alpha$ quanta. This is does not exclude the possibility that at sufficiently large length scales or low temperatures the $b$ and $f^\alpha$ quanta are actually confined.

This paper summarizes recent work[5,6] which studies the consequences of extending the assumption of spin-charge separation from the normal to the superconducting phase. (Some of the discussion below is taken from Ref 5, but additional clarifying remarks have been added; the present paper should be read first and Refs 5,6 can be consulted for additional details.) An additional assumption about the origin of superconductivity will be used: the pairing induced by the antiferromagnetic interactions between the spinons will be taken to be the cause of superconductivity. We will therefore be interested in the pairing amplitude

$$\Delta_{ij} = \left\langle \varepsilon^{\alpha\beta} f^\dagger_{i\alpha} f^\dagger_{j\beta} \right\rangle \tag{2}$$

which measures the tendency of spinons at site $i$, $j$ to form a spin-singlet. A large value of $\Delta$ indicates the presence of strong antiferromagnetic spin-correlations, but not necessarily the presence of superconducting coherence.

This paper shall review how the above assumptions can be used to formulate a phenomenological model $F$ of the superconducting phase and its transition to the anomalous normal state; a closely related phenomenological free energy for superconductors with broken time-reversal invariance has been discussed by Wen and Zee[4] - time-reversal invariance will be assumed to be preserved in the present paper. We will then explore whether the properties of $F$ are in any way distinct from the usual phenomenological Landau-Ginzburg description in terms of a charge $2e$ superconducting order parameter. The main new result will be the existence of parameter regimes in which the lowest energy mechanism for magnetic flux to pierce the system is with vortices carrying flux $hc/e$. Magnetic flux can also penetrate in vortices with flux $hc/(2e)$ but such configurations are found to be not always globally stable as they can lead to a large loss in the antiferromagnetic correlation energy. In contrast, the configurations with $hc/e$ vortices are able to allow penetration of magnetic flux by loss of superconducting coherence in the vortex cores, without a concomitant loss in antiferromagnetic correlations. A microscopic, symplectic large-$N$ expansion[7,8] of a model of the $CuO_2$ layers[6] suggests that the region of stability of the $hc/e$ vortices is the low-doping boundary of the superconducting state - *i.e.* the superconducting region closest to the half-filled insulating state. We argue that this conclusion is also supported by differences between NMR experiments on the normal state in the small and large doping regions[9]. However, a strong first-order superconductor-normal transition could preempt the existence of stable $hc/e$ vortices. Flux decoration experiments of these "low" $T_c$ superconductors will therefore be of great interest.

An important property of the model of this paper is that the preference for $hc/e$ vortices is purely energetic. The fundamental 'flux-quantum' remains at $hc/2e$. In particular, experiments which examine periodicities as a function of a *varying* magnetic field observe a period in total magnetic flux of $hc/2e$ throughout the superconducting phase (see Section 4.2). One such experiment is that of Little and Parks[10] which measures shifts in $T_c$ of a thin-walled superconducting cylinder in an axial magnetic field.

## 2. PHENOMENOLOGICAL FREE ENERGY

We begin by obtaining the phenomenological model, $F$, of superconductivity in the presence of spin-charge separation. We noted above that an important field is the pairing amplitude $\Delta$ of two spinons. However condensation of $\Delta$ is not sufficient to obtain superconductivity; as is well known, and is also shown below to be a simple consequence of $F$, superconductivity requires in addition the condensation of the holon $b$. In contrast to earlier assertions[11], we have shown elsewhere[6] the condensation of single $b$ quanta (and not just pairs of $b$ quanta) occurs in the presence of incommensurate spin-correlations - incommensurate correlations have recently been observed in neutron scattering experiments[12]. The phenomenological free energy, $F$, will therefore be expressed in terms of the condensates of the spinon pairing amplitude $\Delta$ and the holon $b$.

The form of the phenomenological free energy controlling fluctuations of the fields $\Delta$ and $b$ is essentially dictated by gauge invariance. The decomposition (1) of the physical hole operator introduces a redundancy in the degrees of freedom which can be removed by demanding that all observable correlations functions be invariant under the following gauge transformations:

$$\begin{aligned} f^\dagger &\rightarrow f^\dagger \exp(i\chi) \\ \Delta &\rightarrow \Delta \exp(2i\chi) \end{aligned}$$

$$\begin{aligned} b &\rightarrow b \exp(-i\chi - ie\omega) \\ d^\dagger &\rightarrow d^\dagger \exp(-ie\omega) \\ \vec{A} &\rightarrow \vec{A} - \vec{\nabla}\omega \end{aligned} \tag{3}$$

Here $\chi$ generates the internal $U_I(1)$ gauge symmetry introduced by the decomposition (1). The electromagnetic gauge symmetry, $U_{em}(1)$, is generated by $\omega$ and its vector potential is $\vec{A}$. We have absorbed a factor of $1/(\hbar c)$ in the magnitude of $e$. The $d$ fermion carries electromagnetic charge $e$ which is carried completely by the holon $b$. The physical superconducting order parameter $\Psi^{SC}_{ij} = \left\langle \varepsilon^{\alpha\beta} d^\dagger_{i\alpha} d^\dagger_{j\beta} \right\rangle$ is of course invariant under $U_I(1)$ and transforms like a charge $2e$ scalar field $\Psi^{SC} \rightarrow \Psi^{SC} \exp(-2ie\omega)$ under $U_{em}(1)$. In writing down the above gauge transformations we have taken the continuum limit of a lattice gauge-invariance at the wavevector $\mathbf{k} = 0$. In some microscopic models, fluctuations at the antiferromagnetic wavevector $\mathbf{k} = (\pi, \pi)$ are also important; the consequences of including these will be discussed in Section 6.

The assumption of spin-charge separation requires that we allow the $b$, $f^\alpha$ and $\Delta$ quanta to propagate independently, *i.e.* the free energy $F$ contain quadratic spatial-gradient terms in these fields. However the gauge invariances in (3) make it impossible to write down such terms using these fields alone. It is necessary to introduce a gauge connection $\vec{a}$ for the internal gauge symmetry $U_I(1)$ to allow such propagation[2,3,4]; this gauge connection also appears naturally out of microscopic large $N$ expansions[6]. We therefore have the additional transformation rule

$$\vec{a} \rightarrow \vec{a} - \vec{\nabla}\chi \tag{4}$$

In, or close to, the superconducting phase we expect that the fermions can be safely integrated out and the system described solely in terms of $\Delta$, $b$ and $\vec{a}$. The invariances (3), (4) dictate that their action for static fluctuations be of the following form

$$\begin{aligned} F = \int d^2r \Big[ &|(\vec{\nabla} + 2i\vec{a})\Delta|^2 + r_1|\Delta|^2 + \frac{u_1}{2}|\Delta|^4 \\ &+ |(\vec{\nabla} - i\vec{a} - ie\vec{A})b|^2 + r_2|b|^2 + \frac{u_2}{2}|b|^4 \\ &+ v|b|^2|\Delta|^2 + \frac{1}{8\pi}(\vec{\nabla}\times\vec{A})^2 + \frac{\sigma}{2}(\vec{\nabla}\times\vec{a})^2 + \cdots \Big] \end{aligned} \tag{5}$$

with $u_1, u_2 > 0$, and $v^2 < u_1 u_2$. The fields $\Delta$ and $b$ have been rescaled to make the coefficients of their gradient terms unity. The parameters $r_1$, $r_2$, $u_1$, $u_2$, $v$, and $1/\sigma$ all have the dimensions of $E/L^2$ ($L$ is the unit of length, and $E$ the unit of energy) and are expected to be of roughly the same order of magnitude; an exception to this is the region close to the superconductor-normal phase boundary when a combination determining the superconducting coherence length will become large. The term proportional to $\sigma$ represents the 'diamagnetic' response of the spinons that have been integrated out: this is the energy associated with introducing an additional 'flux' into the ground state of the antiferromagnet at half-filling. The electric charge $e$ has dimensions of $\sqrt{E}/L$; we will study only strong type II superconductors, in which case $4\pi e^2 \ll u_1, u_2, 1/\sigma$. It is the inequality $4\pi e^2 \ll 1/\sigma$ which distinguishes the role of $U_{em}(1)$ and $U_I(1)$: it implies that the fluctuations of $\vec{A}$ are almost pure gauge while the flux $\vec{\nabla}\times\vec{a}$ is strongly fluctuating. The connection between $F$ and a symplectic large $N$ expansion[6] on a realistic microscopic model of the $CuO_2$ layers was discussed in Ref 5 and will not be duplicated here; this analysis will give some information on the variation of the parameters in $F$ with temperature and doping.

A cross-term $(\vec{\nabla}\times\vec{a})\cdot(\vec{\nabla}\times\vec{A})$ in $F$ is also permitted by the gauge symmetries of (3), (4). We shall assume that the coefficient of such a term is 0; this is in fact equivalent to the assumption that all of the charge of the hole resides on the holon $b$. A non-zero coefficient of $(\vec{\nabla}\times\vec{a})\cdot(\vec{\nabla}\times\vec{A})$ leads to partial separation of spin and charge. Such a possibility was examined in Ref 5 and shown not to significantly modify any of the conclusions below.

We also introduce the gauge invariant currents

$$\begin{aligned} \vec{J}_\Delta &= \frac{1}{i}\mathrm{Im}\left(\Delta^*(\vec{\nabla}+2i\vec{a})\Delta\right) \\ \vec{J}_b &= \frac{1}{i}\mathrm{Im}\left(b^*(\vec{\nabla}-i\vec{a}-ie\vec{A})b\right) \end{aligned} \qquad (6)$$

Upon examining variations of $F$ with respect to $\vec{A}$, the electromagnetic supercurrent is easily seen to be $\vec{J}_{em} = -e\vec{J}_b$. Stationarity of $F$ with respect to variations in $\vec{a}$ leads to the condition

$$-2\vec{J}_\Delta + \vec{J}_b = \sigma\vec{\nabla}\times(\vec{\nabla}\times\vec{a}) \qquad (7)$$

This equation can be interpreted as the consequence of the local constraint on the spinons and holons; the terms on the left-hand side represent the current of pairs of $f^\alpha$ fermions and the boson current respectively, while the right hand side is the current of the the single $f^\alpha$ fermions which have been integrated out.

## 3. PHASE-DIAGRAM OF $F$

We now discuss qualitative features of the phases of $F$ in the simplest mean-field theory which ignores the fluctuations of the gauge fields. The results of a minimization of $F$ with respect to the mean field values $\Delta = \bar{\Delta}$ and $b = \bar{b}$ are shown in Fig 1 as a function of $r_1$ and $r_2$. At the mean-field level, the point $r_1 = 0$, $r_2 = 0$ behaves like a tetracritical point[13] with four regions converging upon it. These four regions are characterized by finite or zero values of $|\bar{\Delta}|$ and $|\bar{b}|$; the existence of these four regions was also noted by Wen and Zee[4]. We discuss the four regions, and the nature of the boundaries between them, in turn:

(i) *Superconductor*:

Only the region in which both $|\bar{\Delta}|$ and $|\bar{b}|$ are non-zero is superconducting as $\Psi^{SC} \sim \bar{\Delta}\bar{b}^2$. All other regions are "normal" and do not display a Meissner effect for $\vec{A}$. The boundaries between the superconductor and its neighboring regions

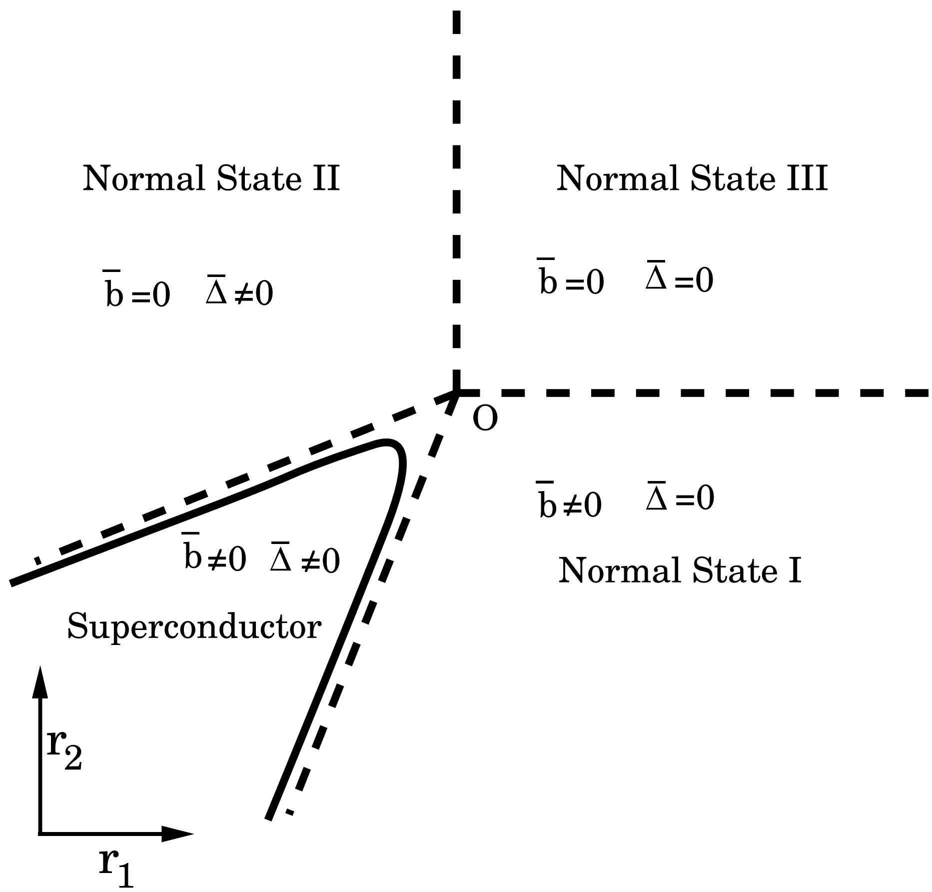


Fig. 1. Mean-field phase diagram of $F$ as a function of $r_1$ and $r_2$ for $v > 0$ and $v^2 < u_1 u_2$. The point $O$ is $r_1 = 0$ and $r_2 = 0$. The mean-field transitions are shown by dashed lines. The boundaries of the superconducting phase are given by $r_1 - (r_2 v)/u_2 = 0$ and $r_2 - (r_1 v)/u_1 = 0$. The expected location of the superconductor-normal transition in the presence of fluctuations is shown by the solid line; the various normal states have only quantitative differences in their properties and are expected to be connected by smooth crossovers in $d = 2$. The region of stability of the $hc/e$ vortices is close to the superconductor-normal-state II phase boundary and well away from the superconductor-normal-state I boundary.

are thus true phase transitions. The normal phases however display important quantitative differences in their properties.
(ii) *Normal state I*:
This is the region with only $\bar{b}$ non-zero and is most like a conventional Fermi liquid. It was shown in Ref 5 that the transition between this region and the superconducting phase is well described by the fluctuations of a scalar, $\Psi_{2e}$, which is invariant under $U_I(1)$ and carries electromagnetic charge $2e$.
(iii) *Normal state II*:
Here only $\bar{\Delta}$ is non-zero and the normal state has strong antiferromagnetic correlations. This is expected to lead to a pseudo-gap feature in the $f^{\alpha}$ fermion spectrum and a suppression of the spin susceptibility. The transition between this phase and the superconductor was shown in Ref 5 to be controlled by the fluctuations of a scalar, $\Psi_e$, which is invariant under $U_I(1)$ and carries electromagnetic charge $e$. We expect $F$ to display a smooth crossover in the superconductor-normal transition between regimes dominated by fluctuations of scalars with charge $e$ and $2e$ as one passes from normal state II to normal state I.
(iv) *Normal state III*:
Now both the mean-field values $\bar{b}$ and $\bar{\Delta}$ are zero. The novel properties of this region have already been examined by Nagaosa and Lee and Ioffe and Weigmann[3].

The consequences of gauge-field fluctuations upon the transitions between normal states I,II, and III are expected to be significantly different from those between the superconductor and the normal states. The superconducting order will be coherent between the $CuO_2$ layers and the critical fluctuations near the superconductor-normal transition will be three-dimensional. In contrast the $U_I(1)$ gauge connection can only be defined within each layer; fluctuations between the normal states are therefore described by a two-dimensional Abelian Higgs model which is expected to possess a smooth crossover and not a phase transition[14]. The non-local order parameter construction[15], which demands the existence of a phase transition between the Higgs and normal phases, fails in $d = 2$. Of course, none of the above considerations rule out a first-order transition between the normal states.

NMR data of the $Cu$ Knight shift in $YBa_2Cu_3O_{6.5+\delta}$ for $\delta \sim 0.1$[9] shows a strong temperature dependent suppression of the spin susceptibility at temperatures above the superconducting $T_c$. This is consistent these compositions and temperatures being identified as normal state II. At larger dopings near $\delta \sim 0.5$, the spin susceptibility of the non-superconducting phase is temperature independent, consistent with the properties of normal state I. I am grateful to A. Millis for drawing my attention to this data. Finally, the normal state III region is expected to appear at higher temperatures at all doping concentrations.

These assignments are also consistent with the results of a previous microscopic large $N$ calculation on a three-band model of the $CuO_2$ layers[6]; this results of this calculation are summarized in Fig 2. Note that the overall topology of the phases is consistent with the Landau theory results summarized in Fig 1; the control parameters $r_1, r_2$ have now been replaced by the temperature $T$ and the doping $\delta$.

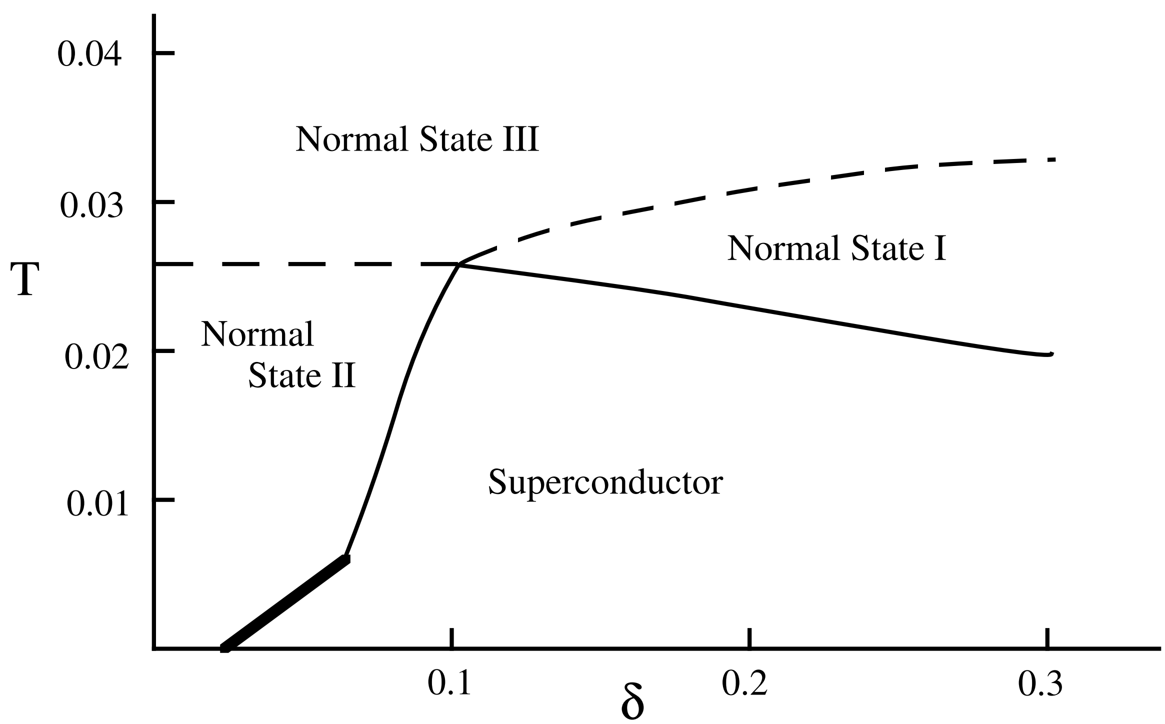


Fig. 2. Phase diagram from Ref 6 of the symplectic large-$N$ calculation on a model of the copper-oxide layers. The $y$-axis is the temperature and the $x$-axis is the doping. Notice the similarity in the topology of this figure and the Landau theory results of Fig 1. The solid line is the only true phase transition and separates the normal states from the superconductor. The thick line at small doping denotes a first-order transition; elsewhere it is second-order. The region of stability of the $hc/e$ vortices is expected to be the superconductor closest to the normal-state II phase.

Just as was argued in the previous paragraph, we find normal state I in the small doping region, normal state II in the large doping region and normal state III at high temperatures. The transition between normal state II and the superconductor is found to be first-order at the lowest temperatures in the large $N$ limit (Fig 2 and Ref 6).

## 4. PROPERTIES OF THE SUPERCONDUCTING PHASE

In this section we examine the properties of the action $F$ (Eqn (5)) in the superconducting phase. We will focus on the response of the system to an external magnetic field in both bulk and multiply connected geometries.

### 4.1. *Electromagnetic Response in the Bulk Superconductor*

Deep within the superconducting phase, it is permissible to replace $\Delta$ and $b$ by the mean-field values $\bar{\Delta}$ and $\bar{b}$ which minimize $F$:

$$\begin{aligned} |\bar{\Delta}|^2 &= -\frac{r_1 u_2 - r_2 v}{u_1 u_2 - v^2} \\ |\bar{b}|^2 &= -\frac{r_2 u_1 - r_1 v}{u_1 u_2 - v^2} \end{aligned} \tag{8}$$

Inserting this into $F$ (Eqn (5)), the resulting action for $\vec{a}$ and $\vec{A}$ takes the form

$$F_a = \int d^2 r \left[ (\vec{a} + e\vec{A})^2 |\bar{b}|^2 + 4\vec{a}^2 |\bar{\Delta}|^2 + \frac{1}{8\pi} (\vec{\nabla} \times \vec{A})^2 + \frac{\sigma}{2} (\vec{\nabla} \times \vec{a})^2 \right] \tag{9}$$

We may now integrate out the massive $\vec{a}$ fluctuations and obtain the following effective action for the electromagnetic field for small $e^2$

$$F_{em} = \int d^2 r \left[ \frac{1}{8\pi} \left( (\vec{\nabla} \times \vec{A})^2 + \frac{1}{\lambda^2} \vec{A}^2 \right) \right] \tag{10}$$

The London penetration depth $\lambda$ is given by

$$\frac{1}{\lambda^2} = 8\pi e^2 \frac{1}{1/|\bar{b}|^2 + 1/|2\bar{\Delta}|^2} \tag{11}$$

Notice that the inverse-square London penetration depth, or equivalently the superfluid density is approximately proportional to the *smaller* of $|\bar{b}|^2$ and $\bar{\Delta}|^2$. In the event that either of them vanishes, so does the superfluid density and the Meissner response. This also demonstrates our earlier assertion that condensation of both $\Delta$ and $b$ is required for the presence of superconductivity.

### 4.2. *Little-Parks Experiment*

In this section we determine the value of the 'flux-quantum', as determined by a Little-Parks experiment[16]. We will find that it takes the value $hc/2e$ throughout the superconducting phase.

Consider a thin-walled superconducting cylinder of radius $R$ with electromagnetic flux $\tilde{\Phi}_A$ along the axis of the cylinder. Ignoring the radial dependencies, we expect that the fields will take the values

$$\begin{aligned} \Delta &= \bar{\Delta}\exp(in_\Delta\theta) \\ b &= \bar{b}\exp(in_b\theta) \\ A_\theta &= \frac{\tilde{\Phi}_A}{2\pi R} \\ a_\theta &= \frac{\tilde{\Phi}_a}{2\pi R} \end{aligned} \tag{12}$$

where $\theta$ is the angular co-ordinate, and the integers $n_\Delta, n_b$ and the real number $\tilde{\Phi}_a$ must be chosen to minimize the value of $F$ in the presence of the electromagnetic flux $\tilde{\Phi}_A$. Inserting (12) into $F$ we find that the free-energy density, $F_R$, is

$$F_R = \frac{|2\bar{\Delta}|^2}{4\pi^2R^2}\left(\pi n_\Delta + \tilde{\Phi}_a\right)^2 + \frac{|\bar{b}|^2}{4\pi^2R^2}\left(2\pi n_b - \tilde{\Phi}_a - \frac{e}{\hbar c}\tilde{\Phi}_A\right)^2 + \ldots \tag{13}$$

where we have reinserted factors of $\hbar c$ and the omitted terms are independent of the fluxes and the phase windings $n_\Delta, n_b$. Finally, we minimize $F_R$ with respect to $\tilde{\Phi}_a$ and obtain

$$F_R = \frac{\pi^2 e^2}{h^2c^2R^2}\frac{1}{1/|\bar{b}|^2 + 1/|2\bar{\Delta}|^2}\left(\tilde{\Phi}_A - \frac{hc}{2e}(n_\Delta + 2n_b)\right)^2 \tag{14}$$

Two important features of $F_R$ are immediately apparent: ($i$) the minimum value of $F_R$ over the set of integers $n_\Delta, n_b$ is a periodic function of $\tilde{\Phi}_A$ with period $hc/2e$; ($ii$) the amplitude of the oscillation is proportional to the superfluid stiffness, or equivalently, the inverse London penetration depth squared (See Eqn (11)).

## 5. VORTICES IN THE SUPERCONDUCTING PHASE

We finally turn to a discussion of the structure of vortices of $F$ in the superconducting phase. It is of course important to characterize the vortex by gauge-invariant quantities. Far from the core of the vortex, finiteness of the energy demands the configuration

$$\Delta(\vec{r}) = \bar{\Delta}\exp\left(i\phi_\Delta(\vec{r})\right) \quad ; \quad b(\vec{r}) = \bar{b}\exp\left(i\phi_b(\vec{r})\right) \tag{15}$$

The values of the phases $\phi_\Delta, \phi_b$ are non-gauge-invariant, but the integers $n_\Delta, n_b$

$$n_\Delta = \frac{1}{2\pi}\oint_C \vec{\nabla}\phi_\Delta \cdot d\vec{r} \quad ; \quad n_b = \frac{1}{2\pi}\oint_C \vec{\nabla}\phi_b \cdot d\vec{r} \tag{16}$$

(where the contour $C$ encircles the core of the vortex) are invariant under non-singular gauge transformations of both $U_I(1)$ and $U_{em}(1)$. Singular gauge transformations for $U_I(1)$ are forbidden by the presence of the $\sigma(\vec{\nabla}\times\vec{a})^2$ term in $F$. Each pair of integers $(n_\Delta, n_b)$ thus defines a topologically distinct vortex configuration. The existence of a two-parameter family of vortices has already been pointed out by Wen and Zee[4]. To determine the values of the fluxes of $\vec{a}$ and $\vec{A}$, we apply the usual argument[17] for the finiteness of the vortex energy to the two gradient terms in $F$. This yields the constraints (after restoring factors of $\hbar c$)

$$\begin{aligned} -2\int d^2r(\vec{\nabla}\times\vec{a})_z &= 2\pi n_\Delta \\ \int d^2r\left[(\vec{\nabla}\times\vec{a})_z + \frac{e}{\hbar c}(\vec{\nabla}\times\vec{A})_z\right] &= 2\pi n_b \end{aligned} \tag{17}$$

for a vortex in the $x, y$ plane. Solving for the total electromagnetic flux we find

$$\int d^2r(\vec{\nabla}\times\vec{A})_z = \frac{hc}{2e}(n_\Delta + 2n_b). \tag{18}$$

Note that, in contrast to the conventional Abrikosov theory, the electromagnetic flux does not uniquely define a vortex configuration; there is an infinite number of choices of the integers $n_\Delta, n_b$ for a given e.m. flux.

We will now estimate the energy of various vortex configurations as a function of the Landau parameters $r_1, r_2$; all other Landau parameters will be assumed to be fixed at values of order unity. The energy $F_v$ can be estimated as the sum of two physically distinct contributions:

$$F_v = F_c + F_{sf} \tag{19}$$

The first term $F_c$ is the core contribution from the region which is within a superconducting coherence length, $\xi$, of the center of the vortex. Its magnitude will estimated below for some illustrative values of $n_\Delta, n_b$. The second term, $F_{sf}$, is the contribution of the region well away from the core of the vortex where the energy is dominated completely by the kinetic energy of the superflow. Under such conditions the properties of $F$ can be shown[5] to be indistinguishable from those of the conventional Landau-Ginzburg free energy of superconductivity. A simple extension[5] of the standard calculation shows

$$\begin{aligned} F_{sf} &\sim \frac{(n_\Delta + 2n_b)^2}{e^2\lambda^2}\ln\kappa \\ &\sim (n_\Delta + 2n_b)^2 \min(|\tilde{r}_1|, |\tilde{r}_2|)\ln\kappa \end{aligned} \tag{20}$$

where $\kappa = \lambda/\xi$ is the Ginzburg-Landau parameter. In the second expression we have used the results (8) and (11) near the boundary between the superconductor and the normal states; $\tilde{r}_1 = r_1 - (r_2 v)/u_2$ is the renormalized "mass" of the $\Delta$ field and equals the horizontal distance to the superconductor-normal state I boundary in Fig 1 while $\tilde{r}_2 = r_2 - (r_1 v)/u_1$ is the renormalized $|b|$ "mass" and equals the vertical distance to the superconductor-normal state II phase boundary. Note that as usual $F_{sf}$ is proportional to the square of the electromagnetic flux in the vortex.

For a given total flux therefore, $F_{sf}$ will be minimized by configurations in which the flux is split into vortices carrying the smallest allowable unit of $hc/2e$.

We now estimate the value of $F_c$ for two elementary vortex configurations:
$(a)$ $n_\Delta = 1$, $n_b = 0$
The flux in this vortex is $hc/2e$. The existence of a non-trivial winding in the phase of $\Delta$ and finiteness of $F$ demand that $|\Delta|$ vanish at the core of the vortex. In contrast, $|b|$ can remain finite at the center. The vanishing of $\Delta$ implies that the system loses both superconducting coherence and antiferromagnetic correlations at the core of the vortex. Standard techniques[17] can be used to estimate the core energy and we find

$$F_c \sim |\tilde{r}_1| \tag{21}$$

$(b)$ $n_\Delta = 0$, $n_b = 1$
The flux in this vortex is $hc/e$. The existence of a non-trivial winding in the phase of $b$ and finiteness of $F$ now demand that $|b|$ vanish at the core of the vortex, while $|\Delta|$ can remain finite. The vanishing of $b$ implies that the system loses superconducting coherence but the finite value of $\Delta$ indicates that antiferromagnetic correlations are preserved. As above, the core energy is estimated to be

$$F_c \sim |\tilde{r}_2| \tag{22}$$

From Eqns. (19), (20), (21), (22) we see that a remarkable situations can develop in the parameter regime

$$\frac{|\tilde{r}_2|}{|\tilde{r}_1|} < \frac{1}{\ln \kappa} \tag{23}$$

The energy of the $hc/2e$ vortex ($n_\Delta = 1$, $n_b = 0$) is dominated by the core contribution and scales linearly with $|\tilde{r}_1|$. The loss of antiferromagnetic correlations in the core of this vortex has a large energy cost in this regime. In contrast the energy of the $hc/e$ ($n_\Delta = 0$, $n_b = 1$) scales linearly with the smaller $|\tilde{r}_2|$. Antiferromagnetic correlations are preserved in the core of this vortex and the system has to only pay the small cost of the loss of superconducting coherence. Placing the superconductor in an external magnetic field larger than $H_{c1}$, under conditions in which (23) is satisfied, will therefore lead to the appearance of vortices with flux $hc/e$. Detailed numerical calculations of the vortex solutions of $F$ have been performed[5] and the region of stability of the $hc/e$ vortices was found to be roughly consistent with (23).

An experimental test of the appearance of $hc/e$ vortices will clearly be useful. It is of course necessary to search for a regime in which $|\tilde{r}_1| \gg |\tilde{r}_2|$. *This is most likely in the region closest to the superconductor-normal state II phase boundary and well away from the superconductor-normal state I phase boundary.* From Figs 1 and 2 and our earlier discussion of the NMR experiments and the large $N$ expansion, we conclude that the most favorable regime is near the low doping onset of superconductivity as one moves away from the insulating state. A strong first-order transition between the superconductor and normal-state II could however prevent the existence of a region in which $|\tilde{r}_1|/|\tilde{r}_2|$ is large enough; the large-$N$ calculation did find this transition to be first-order at the very lowest temperatures (Fig 2).

## 6. VORTICES AND STAGGERED GAUGE INVARIANCE

Some earlier studies[11,18,6] of antiferromagnetically induced superconductivity have attached much importance to the two-sublattice structure of the $CuO_2$ layers. In particular in the presence of strong antiferromagnetic pairing, these theories find two species of holons, each residing on one of the sublattices. In this section, we will extend our results to include the sublattice structure. The main result will be an understanding of the importance of incommensurate spin correlations on the stability of $hc/e$ vortices.

To include the sublattice structure, we have to be more careful in taking the continuum limit of the lattice gauge symmetry. In addition to the uniform component, $U_I(1)$, of the internal gauge symmetry, we need to keep track of its staggered component $U_s(1)$. Under $U_s(1)$ the holons on the two sublattices have opposite charges[6]. Labeling the holons on the two sublattices $b_A$ and $b_B$, we generalize the gauge transformations of (3) to

$$\begin{aligned} b_A &\rightarrow b_A \exp(-i\chi - i\rho - ie\omega) \\ b_B &\rightarrow b_B \exp(-i\chi + i\rho - ie\omega) \end{aligned} \quad (24)$$

where the field $\rho$ generates the staggered gauge transformation $U_s(1)$. The antiferromagnetic pairing amplitude, $\Delta$, involves spinons on opposite sublattices

$$\Delta = \left\langle \varepsilon^{\alpha\beta} f^{\dagger}_{A\alpha} f^{\dagger}_{B\beta} \right\rangle$$

(where $f^{\alpha}_A$, $f^{\alpha}_B$ are the spinons on the $A$ and $B$ sublattices) and therefore does not carry any $U_s(1)$ charge:

$$\Delta \rightarrow \Delta \exp(2i\chi). \quad (25)$$

We also need a field, $\psi$, to allow hopping of the spinons and holons between the two sublattices: we have

$$\psi \sim \left\langle b^{\dagger}_A b_B \right\rangle \qquad \text{or} \qquad \psi \sim \left\langle f^{\dagger}_{B\alpha} f^{\alpha}_A \right\rangle \quad (26)$$

Condensation of $\psi$ implies the appearance of incommensurate spin-correlations[7,8,6]. As such correlations have been observed experimentally[12], we will assume that $\psi$ is condensed over the entire low-temperature region. Under the gauge transformations

$$\psi \rightarrow \psi \exp(2i\rho). \quad (27)$$

Finally we need the gauge connections associated with all three gauge symmetries:

$$\begin{aligned} \vec{A} &\rightarrow \vec{A} - \vec{\nabla}\omega \\ \vec{a} &\rightarrow \vec{a} - \vec{\nabla}\chi \\ \vec{a}_s &\rightarrow \vec{a}_s - \vec{\nabla}\rho \end{aligned} \quad (28)$$

Gauge and sublattice symmetries now constrain the effective action for $\psi$, $\Delta$, $b_A$ and $b_B$ into the following form[6]:

$$F_s = \int d^2r \Big[ |(\vec{\nabla} + 2i\vec{a})\Delta|^2 + r_1|\Delta|^2 + |(\vec{\nabla} + 2i\vec{a}_s)\psi|^2 + r_s|\psi|^2$$
$$+ |(\vec{\nabla} - i\vec{a} - i\vec{a}_s - ie\vec{A})b_A|^2 + |(\vec{\nabla} - i\vec{a} + i\vec{a}_s - ie\vec{A})b_B|^2 + r_2(|b_A|^2 + |b_B|^2)$$
$$+ g(\psi^* b_A^\dagger b_B + \text{H.c}) + \frac{1}{8\pi}(\vec{\nabla}\times\vec{A})^2 + \frac{\sigma}{2}(\vec{\nabla}\times\vec{a})^2 + \frac{\sigma'}{2}(\vec{\nabla}\times\vec{a}_s)^2 + \cdots \Big] \quad (29)$$

Higher-order terms which stabilize the action have not been explicitly written down. Fradkin and Kivelson[18] also considered a similar action but without the fields $\Delta, \psi$.

We now examine vortex minima of $F_s$. The existence of superconductivity requires that $\Delta, b_A, b_B$ be condensed. Condensation of $b_A$, $b_B$ will induce condensation of $\psi$ (via the term proportional to $g$ in $F_s$) in the superconducting phase. A superconducting phase with purely commensurate spin correlations is therefore not within the realm of possibilities of the theories considered in this paper. However, for reasons which will become clear below, we will require that $\psi$ be condensed in the normal phase before the onset of superconductivity, and thus $r_s < 0$ at the superconductor-normal phase boundary. Far from the core of the vortex the magnitudes of $\Delta$, $\psi$, $b_A$, and $b_B$ will saturate at constants but their phases may have a non-trivial winding. Let the winding numbers of their phases be respectively $n_\Delta$, $n_\psi$, $n_A$ and $n_B$. Then, a standard argument appealing to the finiteness of the energy shows that

$$\begin{aligned}
\int d^2r \left[(\vec{\nabla}\times\vec{a}) + (\vec{\nabla}\times\vec{a}_s) + \frac{e}{\hbar c}(\vec{\nabla}\times\vec{A})\right] &= 2\pi n_A \\
\int d^2r \left[(\vec{\nabla}\times\vec{a}) - (\vec{\nabla}\times\vec{a}_s) + \frac{e}{\hbar c}(\vec{\nabla}\times\vec{A})\right] &= 2\pi n_B \\
\int d^2r (\vec{\nabla}\times\vec{a}) &= \pi n_\Delta \\
\int d^2r (\vec{\nabla}\times\vec{a}_s) &= \pi n_\psi
\end{aligned} \quad (30)$$

where we have re-inserted factors of $\hbar c$. Consistency among these equations requires that

$$n_\psi = n_B - n_A \quad (31)$$

The total electromagnetic flux is found to be

$$\int d^2r (\vec{\nabla}\times\vec{A}) = \frac{hc}{2e}(n_\Delta + n_A + n_B) \quad (32)$$

The crucial point, apparent from Eqns (31) and (32), is that vortices with flux $hc/2e$ necessarily have

$$\text{either } n_\psi \neq 0, \text{ or } n_\Delta \neq 0 \text{ or both.} \quad (33)$$

However a vortex with flux $hc/e$ with $n_A = 1$ and $n_B = 1$ can have $n_\psi = n_\Delta = 0$. By an argument very similar to that in Section 5 we may conclude that in a region with strong antiferromagnetic correlations ($r_1 \ll 0$), incommensurate spin correlations ($r_s \ll 0$), but weak superconductivity ($r_2 < 0$), vortices with flux $hc/e$ will be stable. Note that *it is crucial that incommensurate spin correlations be present in the normal state before the onset of superconductivity.* Recent experiments[12] show this requirement to be satisfied. Only under such conditions will there be an energy gain associated with vortices which preserve the incommensuration (*i.e.* $\psi \neq 0$) in their cores. Note also that if the spin correlations were commensurate in the normal state, our theory shows that the onset of superconductivity would in any case induce incommensurate correlations; this latter effect is however not sufficient for the stability of $hc/e$ vortices.

## 7. CONCLUSIONS

This paper has examined a phenomenological model of superconductivity in strongly correlated electronic systems[1,3,4]. The strong repulsive interactions introduce a continuum $U_I(1) \times U_s(1)$ gauge invariance which is crucial in restricting the form of the phenomenological free energy. The assumption of spin-charge separation and the gauge invariances require the introduction of gauge-connections to allow for independent propagation of the spinons and holons[2,3]. The superconductor was described by a phenomenological free energy, $F$, expressed in terms of condensates of the holons and the spinon pairing field $\Delta$.

An attempt was then made to determine if $F$ displayed any measurable difference from the usual Landau-Ginzburg free energy of a conventional superconductor. It was found that over a large portion of the superconducting phase, the two approaches were essentially indistinguishable. One striking difference did however appear in the response of the superconductor to an external magnetic field. Near the phase boundary in $F$ between the superconductor and an antiferromagnetically correlated normal state, vortices with flux $hc/e$ generically become the optimum way for the magnetic field to pierce the system. It was shown that such vortices can pierce the system without leading to a significant loss in the antiferromagnetic correlation energy, while $hc/2e$ vortices always have a significant energy loss associated with poor spin correlations in their cores. In the cuprate superconductors the most favorable region for this was found to be the lowest doping concentrations at which superconductivity first occurs. *The $hc/e$ vortices also become increasingly likely as the field goes from $H_{c1}$ to $H_{c2}$*: this is because the vortex core-energy contributes the largest fraction of the total energy at $H_{c2}$. An experimental search for such vortices will be quite useful. An important caveat is that the $hc/e$ vortices could be preempted by a strong first-order transition between the superconductor and the normal state. In either case, an experimental test of the scenario of this paper is available.

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