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Physica E 20 (2003) 117–122
PHYSICA E
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Radiation-induced magnetoresistance oscillations in a 2D electron gas☆

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Abstract

Remarkable recent experiments have shown that microwave radiation can induce dramatic changes in the DC transport properties of a high mobility two-dimensional electron gas. In particular, the diagonal resistivity is an oscillatory function of ω/ω_c, where ω is the microwave frequency and ω_c is the cyclotron frequency. The amplitude of the oscillations increases with microwave intensity and eventually saturates as the minimum resistivity approaches zero. We describe a simple model, first proposed many years ago by Ryzhii and collaborators, to explain the effect, and present simplified but detailed and non-perturbative calculations of the non-equilibrium response of the electron gas. We also review some of the many different theoretical pictures that have been proposed to explain the physics and discuss open questions which remain.

PACS: 73.40.−c; 78.67.−n; 73.43.−f

Keywords: Two-dimensional electron gas; Microwave; Zero resistance; Cyclotron resonance

1. Introduction

The various quantum Hall effects have been a remarkable testing ground for our understanding of novel correlated many-body states of two-dimensional electron gases (2DEGs) in strong magnetic fields. A surprising new non-equilibrium phenomenon has been discovered in recent experiments [1–6] in which a high-mobility 2DEG is subjected to microwave radiation. It was found that at relatively low magnetic fields (corresponding to Landau level filling factor $\nu \sim 50$), microwaves induce an oscillation in the longitudinal DC resistivity as a function of ω/ω_c. These oscillations appear to be semi-classical in nature since unlike Shubnikov–de Haas (SdH) oscillations, they are not sensitive to the particular value of the chemical potential (filling factor).

According to Ref. [3], the phase of the oscillations is such that the minimum resistance values are obtained near $\omega/\omega_c = \text{integer} + \frac{1}{4}$ and the resistance at the cyclotron resonance frequency is nearly unchanged. These results are surprising since naively one would only expect a peak at $\omega = \omega_c$ due to heating at the cyclotron resonance. In high mobility samples, both Mani et al. [3] and Zudov et al. [4] observed that as the radiation intensity is increased, the amplitude of the oscillations increases and the minimum resistance values saturate as they approach zero. These novel

☆ This paper was presented at EP2DS15, Nara, Japan, July 2003 and at the International Symposium Quantum Hall Effect: Past, Present, and Future, Stuttgart, Germany, July 2003.

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doi:10.1016/j.physe.2003.09.028

experimental discoveries have unleashed a flood of papers discussing many different possible physical mechanisms [7–27].

One might imagine that this zero-resistance state indicates the formation of a new strongly correlated many-body quantum Hall state, but in fact the Hall resistivity behaves classically (as expected at low magnetic fields) and remains largely unaffected by the microwaves. In particular, there is no evidence of quantized Hall plateau formation induced by the application of microwaves. We believe however that the zero-resistance state does not represent a new strongly correlated state of matter, but rather is a manifestation of non-equilibrium dynamics under the influence of the microwaves. Vavilov and Aleiner [9] have recently developed a rather complete and appealing semi-classical description of this non-equilibrium dynamics.

We present here a very simple model [7] which we believe qualitatively explains the existence of the resistance oscillations, and yields the correct period and phase. Long before the discovery of the quantum Hall effect, Ryzhii and collaborators [28,29] predicted from second-order perturbation theory in the microwave amplitude that such microwave-induced oscillations could be observed in thin metal films subjected to a magnetic field. The physical picture of the mechanism we presented in Ref. [7] is very much the same as in Ryzhii's papers (which were unknown to us at the time of submission). We find that for high radiation intensity, the amplitude of the oscillations can grow large enough that the resistivity becomes negative. Andreev et al. [8] have pointed out that negative resistivity implies the existence of instabilities in the current distribution and from this provide a plausible explanation for why the observed resistance saturates at zero rather than going negative. We have performed a non-perturbative treatment (which sums certain classes of diagrams to all orders) of the effect of the microwave field. At high radiation intensity we find evidence for higher-order multi-photon processes in the DC resistivity which have not yet been observed in experiment.

We begin by presenting the physical picture, give a brief outline of the calculations and present our numerical results. We then briefly review the ideas of Andreev et al. on the current distribution instability. Finally, we discuss some of the many open questions which need further study.

2. Physical picture

The essential physical picture is quite simple. We are looking at a non-linear optical effect in which microwave photons alter the linear response to a DC electric field. Absorption of a microwave photon does not change a particle's momentum and from Kohn's theorem [30] we expect that, in the absence of disorder, the only effect of the microwaves is to excite cyclotron motion of the center of mass degree of freedom which will be unobservable in DC transport. We therefore anticipate that even though the samples have high mobility, disorder and the concomitant momentum relaxation will play a central role in the phenomenon.

In the Landau gauge, position in the x direction is determined by momentum in the y direction. A DC electric field applied in the x direction will tilt the Landau levels as shown in Fig. 1. Absorption of a photon will move the electron vertically in the diagram by an energy $\hbar\omega$ but not change its momentum. Disorder can assist the absorption process by giving a momentum kick to the electron which changes its x position by a corresponding amount. Consider for example a process that begins with an electron at the center of a Landau level where the density of states is high. If ω slightly exceeds ω_c then the density of final states is higher if the electron is kicked to the right than to the left. This tends to pump the electron uphill to the right, opposing the steady downhill dissipative drift. Examination of the diagram shows that the left–right asymmetry in the density of states is largest for $\omega = \left(n \pm \frac{1}{4}\right)\omega_c$ (with opposite signs in the two cases) and vanishes for $\omega = n\omega_c$ and $\left(n + \frac{1}{2}\right)\omega_c$.

Consider on the other hand, a process that begins with an electron half way between two Landau levels. Then the left–right asymmetry will be the reverse of that just described and tend to cancel out the effect. However the density of initial states for this case is much lower and so the effect, while reduced is not eliminated. The net resulting left–right asymmetry scales linearly with the tilt of the Landau levels and hence DC electric field. Thus it makes a *finite* correction to the linear response DC conductivity.

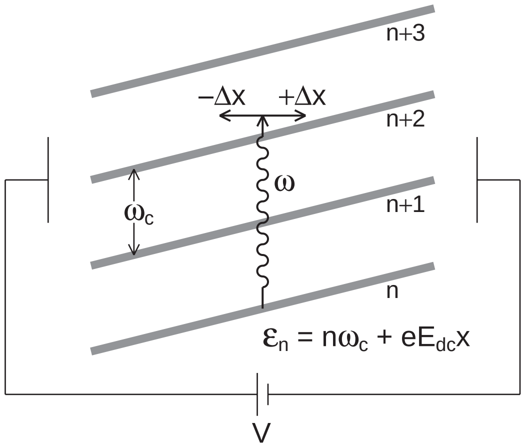

Fig. 1. Simple picture of radiation-induced disorder-assisted current. In the Landau gauge, position in the x direction is determined by momentum in the y direction. Landau levels are tilted by the applied DC bias. Electrons absorb photons and are excited by energy ω. Photo-excited electrons are scattered by disorder and the corresponding momentum change results in a change in x position $\pm\Delta x$. If the final density of states to the left exceeds that to the right, DC current is enhanced. If vice versa, DC current is diminished. Note that electrons initially near the center of a Landau level (where the initial density of states is greatest) will tend to flow uphill for $\omega/\omega_c \approx$ integer $+ \frac{1}{4}$ and downhill for $\omega/\omega_c \approx$ integer $- \frac{1}{4}$. The situation is reversed for electrons initially between two Landau levels, but the initial density of states is lower so these processes do not cancel, and there is a net oscillation of the DC dissipation with microwave frequency.

While the above treatment is highly oversimplified, it indicates that disorder plays a crucial role and may be all that is necessary to obtain the qualitative features of the radiation-induced oscillations. This suggests that a diagrammatic (Kubo formula) calculation [7] of the conductivity, including radiation and disorder but neglecting electron–electron interactions, will be sufficient to reproduce the effect. This calculation and its results are outlined below. We emphasize that while the simplified treatment discussed above focussed on the periodicity of the density of states with energy, the full calculation [7] we outline below also contains other effects, in particular quantum interference terms [21].

3. Calculation

The details of the calculations are presented in Ref. [7]. Here we simply outline the basic idea.

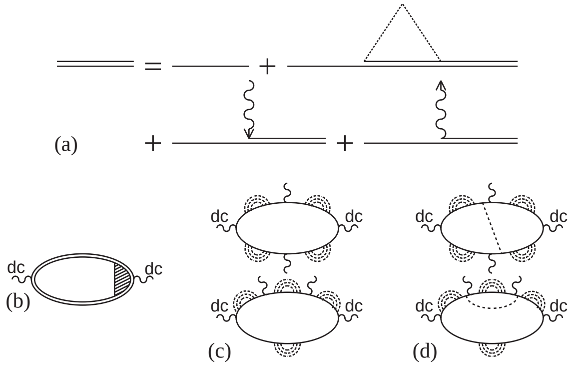

Fig. 2. Diagrams for (a) Green's function and (b) polarization bubble including radiation and disorder within SCBA. Disorder lines are dotted. Photon lines are curvy. The vertex of the polarization bubble is dressed with ladders of disorder lines. When the no-photons-crossed-by-disorder-lines conserving approximation is made, diagrams like those in (c) are included while those in (d) are neglected.

Following the calculation of Ando [31] for the zero-radiation case, we include disorder within the self-consistent Born approximation (SCBA) and assume δ-correlated disorder for simplicity. While somewhat inappropriate for the long-ranged impurity potentials associated with modulation doping, this assumption significantly simplifies the problem by eliminating the momentum dependence of the self-energy. Yet, as we shall see, it manages to capture the important physics rather well. To all orders in the disorder and radiation, the Green's functions are given by the diagrams in Fig. 2(a). The presence of radiation renders this an inherently non-equilibrium problem which requires the non-equilibrium diagrammatic approach of Kadanoff, Baym and Keldysh (see Refs. [32–34] for details). Given the Green's functions, the electrical conductivity is obtained via the Kubo formula, $\sigma_{ij} = -\lim_{\Omega\to 0} \operatorname{Im} \Pi^R_{ij}(\Omega)/\Omega$, where Π^R_{ij} is the retarded current–current correlation function, and $i, j = \{x, y\}$. Diagrammatically, this means evaluating the polarization bubble in Fig. 2(b) where the vertex is dressed by ladders of impurity lines.

We make the simplifying approximation of neglecting vertex corrections to the polarization bubble. To do so within a conserving approximation (one which preserves conservation laws), we must also neglect all diagrams in which impurity lines cross photon insertions (see Fig. 2(c,d)).

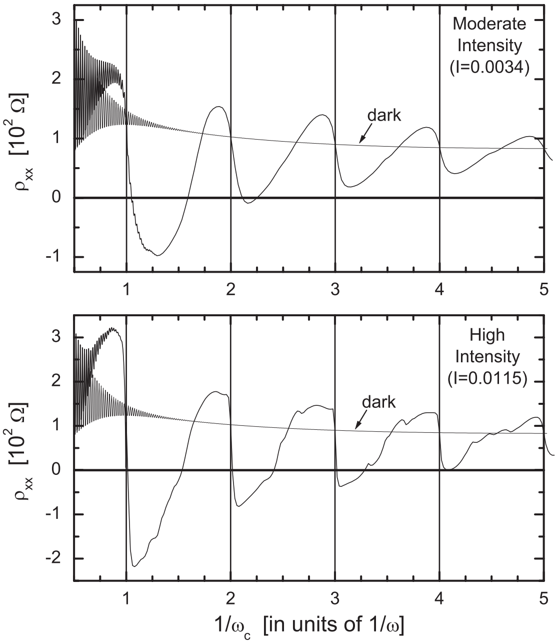

Fig. 3. Calculated radiation-induced resistivity oscillations. We plot ρ_{xx} vs $1/\omega_c$ at fixed ω for $\mu = 50\omega$, $k_BT = \omega/4$, $\gamma = 0.08\omega$, and three values of radiation intensity (in units of $m^*\omega^3$): $I = 0$ (dark), $I = 0.0034$ (upper panel), and $I = 0.0115$ (lower panel). For computational purposes, the energy spectrum is cutoff at 20 Landau levels above and below the chemical potential. The high-frequency oscillations seen at small $1/\omega_c$ are the familiar SdH oscillations with period $1/\mu$.

Within the approximations discussed above, and using parameter values appropriate to the experiments of Ref. [3], we have calculated ρ_{xx} numerically to all orders in radiation and disorder. Results are presented in Fig. 3, where we plot ρ_{xx} versus $1/\omega_c$ for fixed radiation frequency. We consider three values of radiation intensity (power per unit area in units of $m^*\omega^3$): $I = 0$ (dark), $I = 0.0034$ (moderate intensity), and $I = 0.0115$ (high intensity). The dark resistivity exhibits only the familiar SdH oscillations which have period $1/\mu$ and decay away as ω_c becomes small compared with the temperature. For moderate intensity radiation, we find a pronounced radiation-induced oscillation of the longitudinal resistivity. In agreement with experiment, the period of oscillation is $1/\omega$ and minima are found *near* $\omega/\omega_c = \text{integer} + \frac{1}{4}$. We note, however, that the $\frac{1}{4}$ phase shift is not universal, varying between 0 and $\frac{1}{2}$ depending upon disorder and intensity. A more robust feature is the presence of zeros of the oscillation at the integer values of ω/ω_c. Unlike the SdH oscillations, the radiation-induced oscillations are *not* sensitive to the temperature-broadening of the electron distribution about the Fermi level. Temperature dependence therefore derives from the temperature dependence of the scattering, a feature absent from the present calculation due to our neglect of interaction effects. The magnitude of the radiation-induced oscillations can easily exceed the dark resistivity, leading to regions of negative total resistivity. This is reasonable for a non-equilibrium system, and a similar effect has been observed experimentally in semiconductor superlattices [35]. As we discuss further below, some additional physics is required for these negative-resistivity minima to become the zero-resistance states observed in the present experiments. For high intensity radiation, additional features appear in our calculations which have yet to be observed experimentally. These likely correspond to multi-photon processes by which an electron absorbs *m* photons and is promoted by *n* Landau levels.

It is important to note that the simplifying approximations discussed above, employed in order to make the calculation tractable, are not well controlled. As a result, we cannot expect our results to be quantitatively accurate. However, the fact that even this simplified calculation captures the qualitative form of the experimental results encourages us to believe that we have identified the essential physics of the problem. Due to the assumption of δ-correlated disorder, even the zero-radiation (dark) resistivity, ρ^0_{xx}, does not agree quantitatively with experiment. As the calculation neglects the difference between the transport lifetime, τ_{tr}, and the single particle lifetime, τ, it overestimates ρ^0_{xx} by a factor of τ_{tr}/τ, equal to about 50 in the present case. Since the dark Hall resistivity, ρ^0_{xy}, is approximately independent of disorder in the magnetic field regime of interest, the calculated value of ρ^0_{xy} is correct, but effectively too small compared to ρ^0_{xx}. This quantitative error in the dark resistivity has two important consequences for our calculation. When we calculate the radiation-induced change in the Hall resistivity, $\Delta\rho_{xy}$, we find that it is also oscillatory and of comparable magnitude to $\Delta\rho_{xx}$. Since ρ^0_{xy} is effectively too small, we find a total Hall resistivity that exhibits noticeable radiation-induced oscillations. For analogous reasons, while our calculations yield oscillations in σ_{xx} which grow linearly with radiation intensity, the intensity-dependence is muted when we invert the conductivity matrix to obtain ρ_{xx}. However, correcting for the missing factor of 50, it is clear that the effect of radiation on ρ_{xy} is negligible and that the oscillations in ρ_{xx} grow linearly with intensity, which is what was observed experimentally.

4. Current distribution instabilities

While absolute negative resistance under microwave radiation has been observed experimentally in multi-quantum well systems [35], the main feature seen in the present 2DEG systems seems to be a saturation of the resistance minimum at zero. Andreev et al. [8] have proposed an explanation in terms of current instabilities associated with negative resistance. They assume a phenomenological constitutive relation between the electric field and the current density

$$\boldsymbol{E} = \boldsymbol{j}\rho_{xx}(j^2) + \rho_{xy}\boldsymbol{j} \times \hat{z} \qquad (1)$$

in which the Hall resistivity ρ_{xy} is independent of current, but the dissipative resistivity ρ_{xx} starts out negative at low current density and rises through zero at a critical value of the current density j_0 and then becomes positive, asymptotically approaching a value independent of the microwave radiation. Because a spatially uniform current is unstable if $\rho_{xx} < 0$, the current distribution will adjust itself so that the magnitude of the current is close to j_0 everywhere in the sample. The constraint determined by the power supply for the net current flowing through the sample is accommodated by having domains of suitable width some of which have the current flowing opposite to the applied current. Electrodes on the edges of the sample will detect very small voltage drop if the current density is locally close in magnitude to j_0 and this explains the saturation of the apparent net resistance at zero.

It will be very interesting to have experiments which attempt to directly observe the presence of the domain walls. Because the Hall angle is large, the electric field will change sign rapidly across these domain walls which will require a local build up of charge density. It might be possible to detect these electric fields by their effect on the optical response of the semiconductor or directly via scanning probe measurements.

5. Open questions and other physical pictures

The picture we have described appears to qualitatively capture the physics of the microwave-induced zero resistance state, but many important questions remain open. We need to understand the role in this problem of inelastic scattering and energy relaxation due to Coulomb interactions and phonon emission which can be quite complicated in the presence of disorder [36,37]. Ryzhii and collaborators have begun a program in this direction [22,23].

One way around Kohn's theorem that avoids disorder is from edge effects associated with the finite sample size. These can lead to optical absorption at magnetoplasmon frequencies which are considerably shifted from the cyclotron frequency and depend on the sample size. In earlier generations of experiments [38] edge-induced magnetoplasmon resonances were observed, but mysteriously they have *not* been seen in the high-mobility samples used in the recent experiments which are otherwise physically rather similar. It is useful to note that for very high mobility samples, heating probably has a smaller effect on dissipation (since the effect of temperature strictly vanishes for zero disorder). This might explain why the usual resonance signature consisting of a peak in dissipation due to extra heating when the microwave frequency is tuned to the cyclotron or magnetoplasmon frequencies is not observed in these samples.

Mikhailov has correctly emphasized the striking nature of the contradiction with earlier microwave studies and has made an interesting suggestion that high mobility samples may undergo drift plasma instabilities near the edge and has argued that this could also qualitatively explain the experimental results [14].

6. Conclusions

In conclusion, we have presented the simplest possible model of the non-equilibrium dynamics of electrons in a 2DEG in the presence of DC electric and magnetic fields subjected to microwave radiation. In this model the dissipative drift of the electrons is enhanced or reduced due to the combined effect of disorder and the microwaves. The model predicts oscillations in the dissipation as a function of frequency whose phase is consistent with experiment. Our model does not directly treat Coulomb interactions and inelastic relaxation, but simply assumes efficient rethermalization. It also ignores potentially important edge effects.

These new and exciting experimental results clearly deserve considerable further study in order for us to fully understand the novel non-equilibrium dynamics of this system. It would also clearly be desirable to directly search for possible domain structures in the current distribution and to reduce the sample size and pattern the geometry to attempt to control the spatial patterns of domain formation and spatial patterns.

Acknowledgements

We are grateful to K. Lehnert who brought to our attention the experiments of Ref. [35], which helped inspire the physical picture discussed in this Letter. We are also grateful to R. Willett for raising the possibility of negative resistance in these systems. We thank R. G. Mani, R. Shankar, and R. Willett for helpful discussions. This work was supported by NSF Grants DMR-0103639 (A.C.D.), DMR-0098226 (S.S. and A.C.D.), and DMR-0196503 (S.M.G. and A.C.D.).

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