Order and quantum phase transitions in the cuprate superconductors

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Abstract

This is a summary of a review article appearing in Reviews of Modern Physics, July 2003. The ground state of the cuprate superconductors is described using the paradigm of competing orders. This approach has led to numerous predictions, some of which have been tested in recent nanoscale experiments.

1 INTRODUCTION

There is now little doubt that superconducting state of the cuprates has many key features in common with the traditional "low T_c" superconductors, as described by BCS theory [1]. The superfluidity arises from the condensation of Cooper pairs of electrons, and this leads to the characteristic charge $2e$ quantization in the flux quantum and in the Josephson effect. However, when the cuprate superconductors are subjected to nanoscale perturbations, their response deviates strongly from that expected from BCS theory. In particular, BCS theory predicts that when the electron pairing amplitude is locally suppressed, features of the underlying metallic Fermi surface will emerge: this is the physics behind the predictions of BCS theory for the electronic spectrum in the vortex cores, and for the behavior of the superconductor near impurities, boundaries, and defects. It is the author's contention that, in the cuprates, such experiments are better understood using the paradigm of *competing orders.* In the underdoped regime, there is no Fermi liquid proximate to the cuprate superconductor, but a Mott insulator. Order parameters characterizing this Mott insulator lurk just beneath the predominant pairing order of BCS theory, and can reveal themselves under the influence of external perturbations.

The thesis summarized in the previous paragraph has been reviewed at length in a recent article by the author [2]. Here we highlight the key points, and refer the reader to the full article for more details and complete citations to the literature.

2 Mott insulators

A prerequisite is an understanding of the order of Mott insulators. The Mott insulator La_2CuO_4 has a very simple type of order: a collinear, two-sublattice antiferromagnet. We can describe this (and other) Mott insulator by writing

$$\langle \mathbf{S}_j \rangle = \mathbf{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right) \tag{1}$$

where $\mathbf{S}_j$ is the spin operator on site j, $\vec{r}_j$ is the spatial location of the site j, $\vec{K}$ is the ordering wavevector, and $\mathbf{N}_{1,2}$ are two fixed vectors in spin space. The state is La_2CuO_4 has $\vec{K} = (\pi, \pi)$. More generally, (1) applies also to the lightly doped cuprates, where $\vec{K} \neq (\pi, \pi)$. For such values of $\vec{K}$ we need to also distinguish the relative orientations of the ordering wavevectors $\mathbf{N}_{1,2}$. In the cuprates, all experimental indications so far are that the spins are *collinear i.e.* $\mathbf{N}_1 \times \mathbf{N}_2 = 0$. Mott insulators on more frustrated lattices (such as the triangular lattice) can have non-collinear spins, with $\mathbf{N} \times \mathbf{N}_2 \neq 0$.

Apart from Mott insulators which break spin rotation invariance, it is also essential to understand the order in paramagnetic Mott insulators which preserve the spin symmetry of the Hamiltonian with $\langle \mathbf{S}_j \rangle = 0$. One approach to understanding these is to study a quantum phase transition induced by the fluctuations of $\mathbf{N}_1$ and $\mathbf{N}_2$; this ultimately leads to phase in which they have a vanishing static average value. This approach leads to paramagnetic Mott insulators with two distinct types of order:

- The first class has confined spinons and *bond order.* There are no elementary neutral $S = 1/2$ excitations (spinons) because these are permanently confined in pairs: the lowest excitation with a non-zero spin is a $S = 1$ collective mode. The bond order is associated with a modulation in the ground state expectation value of the variable

$$Q_a(\vec{r}_j) \equiv \mathbf{S}_j \cdot \mathbf{S}_{j+a} \tag{2}$$

which breaks the lattice symmetry of the underlying Hamiltonian in a manner which ensures that the new unit cell has an even number of $S = 1/2$ degrees of freedom. This can be loosely viewed as the crystallization of the singlet valence bonds between the spins into some regular arrangement. One important consequence of the confinement of spinons in this state is that a non-magnetic impurity (which removes a single $S = 1/2$ spin from the lattice) will trap a free $S = 1/2$ moment in its vicinity. Finally, this confining, paramagnetic Mott insulator is generically proximate to a state with *collinear* magnetic order.

- The second class of paramagnetic Mott insulators has free spinons and *topological order*, and is generically proximate to a state with *non-collinear* magnetic order. The topological order characterizes a subtle 'rigidity' of the ground state, associated with the pairing of the spins into singlet bonds. Although the singlet bonds form a liquid-like state, any line drawn across the lattice has a definite parity determined by whether it intersects an even or odd number of singlet bonds. This Z_2 quantum number leads to a ground state degeneracy on multiply connected lattices, and also allows new vortex-like defects.

3 Order in doped Mott insulators

Given the experimental observation of collinear magnetic order in the doped cuprates, and the above classification of Mott insulators, a natural hypothesis is that the cuprates are described by the following set of competing orders: (*i*) the pairing order of BCS theory associated with condensation of Cooper pairs, (*ii*) collinear magnetic order as in (1) with $\mathbf{N}_1 \times \mathbf{N}_2 = 0$, and (*iii*) bond order produced by modulations of the expectation value of the Q_{ij} in (2) which break lattice symmetries. Microscopic calculations, quantum field theories, and phenomenological considerations lead to phase diagrams with ground states characterized by long-range correlations in one or more of these orders. The nature of the quantum critical points separating such states can also be studied in some detail.

Of particular importance are the phase boundaries surrounding the state with only the long-range order of the BCS state. These separate the superconductor from states which have long-range correlations in a competing order. We can use the quantum critical theory of such phase boundaries to expand back towards the BCS state, and so obtain a systematic theory for the regime in which the competing order is only 'fluctuating'. Predictions from such an approach have already been subjected to experimental tests, and we survey these below:

- Neutron scattering experiments[3, 4, 5, 6, 7] have explored the interplay between collinear magnetic order and superconductivity in doped La_2CuO_4. In particular, a magnetic field H, applied perpendicular to the layers was used to suppress superconductivity; by the picture outlined above, this should lead to an indirect enhancement of the collinear magnetism. A semi-quantitative analysis of this effect in the vicinity of the quantum critical point was carried in Ref. [8]: it was argued that the magnetic order at a field H and doping δ should be characteristic of the magnetic order in zero field at an *effective* doping $\delta_{\text{eff}}(H)$ given by
$$\delta_{\text{eff}}(H) = \delta - \mathcal{C}\left(\frac{H}{H_{c2}}\right)\ln\left(\frac{H_{c2}}{H}\right), \qquad (3)$$
where $\mathcal{C}$ is some constant and H_{c2} is the upper-critical field for superconductivity. The measurements so far are consistent with the field dependence implied by (3).

- A separate set of nanoscale experiments were undertaken by the Stanford-UBC group [9, 10] to test for the presence of topological order characteristic of the deconfined paramagnetic Mott insulator. Theory predicted [11, 12, 13] an enhanced stability of vortices with flux hc/e and a flux memory effect, but neither of these have been observed so far. This is consistent with the contention that we need only consider orders associated with confining Mott insulators.

- We noted earlier that a non-magnetic impurity (such as Zn on a Cu site in La_2CuO_4) in a confining paramagnetic insulator necessarily induced a spin $S = 1/2$ moment in its vicinity. In a superconducting doped Mott insulator, a likely possibility is that this moment survives down to $T \to 0$, and yields a Curie susceptibility. For large enough doping, however, Kondo screening from the nodal quasiparticles will become strong enough, and there will eventually be an impurity quantum phase transition to a Kondo screened state and a finite $T = 0$ susceptibility [14]. A number of ingenious NMR experiments in the doped cuprates [15, 16] appear to be consistent with just such a behavior.

- Complementary to the neutron scattering studies mentioned above are direct real-space images of the vortex lattice that can be obtained by the scanning tunnelling microscope (STM). The same theory that predicts and enhancement of collinear magnetic spin fluctuations also implied that there should be a 'halo' of pinned bond order around each vortex core [17, 8]. Recent STM studies of the modulations in the local density of states in $Bi_2Sr_2CaCu_2O_{8+\delta}$ appear consistent with this [18, 19, 20], but more detailed studies are probably necessary before this can be considered to have been conclusively established.

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