

Spin and charge order in Mott insulators and d -wave superconductors

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We argue that aspects of the anomalous, low temperature, spin and charge dynamics of the high temperature superconductors can be understood by studying the corresponding physics of undoped Mott insulators. Such insulators display a quantum transition from a magnetically ordered Néel state to a confining paramagnet with a spin gap; the latter state has bond-centered charge order, a low energy $S = 1$ spin exciton, confinement of $S = 1/2$ spinons, and a free $S = 1/2$ moment near non-magnetic impurities. We discuss how these characteristics, and the quantum phase transitions, evolve upon doping the insulator into a d -wave superconductor. This theoretical framework was used to make a number of predictions for STM measurements and for the phase diagram of the doped Mott insulator in an applied magnetic field.

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I. INTRODUCTION

Although there is general agreement that the ground state of the high temperature superconductors is a conventional BCS d -wave superconductor, a number of low temperature properties do not fit easily into this framework, especially in the underdoped regime. Among these are (a) the presence of a $S = 1/2$ moment near non-magnetic Zn or Li impurities in the underdoped superconductor¹⁻³, (b) low energy collective spin excitations (a $S = 1$ spin exciton) at incommensurate wavevectors near (π, π) , and (c) instabilities to various co-existing spin and charge density wave states (“stripes”) in certain materials. While it is certainly possible to devise microscopic models, and a corresponding Hartree-Fock treatment, to generate any of these physical properties in the superconducting state, such a procedure is somewhat ad hoc and has limited predictive power.

Following an early suggestion of Anderson⁴, we explore here the idea that these anomalous properties are related to fundamental characteristics of the undoped Mott insulator, and argue that this leads to a deeper understanding of the physics⁵. The strategy can be summarized by referring to the phase diagram in Fig 1. The first step in our theoretical program is an understanding of paramagnetic Mott insulators which are ‘near’ the Néel state *i.e.* those obtained across a continuous quantum transition at which Néel order is destroyed (this corresponds to moving vertically along the $\delta = 0$ line in Fig 1). There has been much work on this question in the last decade, and we refer the reader to a recent paper by the author⁶ for a discussion of recent progress along with a review of earlier work. The main result we focus on here is the prediction^{12,8} that the appropriate paramagnetic Mott insu-

lator has bond-centered charge order and confinement of spinons. Fig 2 contains a sketch of a likely bond-ordered state, along with a simple physical argument for its selection. A correspondingly simple argument for the confinement of $S = 1/2$ spinons in such a state is sketched in Fig 3. The elementary excitation appears naturally as a $S = 1$ exciton indicated in Fig 3. (This exciton will appear as a ‘resonance peak’ in the neutron scattering cross-section, and the first prediction that such a peak would appear after the loss of magnetic order in *doped* antiferromagnets (with or without spin-Peierls order) was made in Ref. 14.) Another crucial property of the confining paramagnetic insulator is the prediction^{12,2} of a free $S = 1/2$ moment near each non-magnetic impurity: a simple argument for this appears in Fig 4. The reader should already notice a strong similarity between the properties of the paramagnetic Mott insulating state we have just discussed and the properties (a)-(c) of the underdoped superconductor. We will amplify on this connection in the following sections, which contain a review of recent work, accompanied by a discussion of some relevant experiments.

II. NON-MAGNETIC IMPURITIES

We have already mentioned the recent observation of $S = 1/2$ moments near non-magnetic impurities at low T in the underdoped high temperature superconductors by the Orsay group^{1,3}. Specifically, they used NMR measurements to show that the local susceptibility near a non-magnetic Li impurity has a $1/T$ divergence as $T \rightarrow 0$. It is expected that in this regime the impurity susceptibility $\chi_{\text{imp}}(T)$, formally defined as the excess contribution

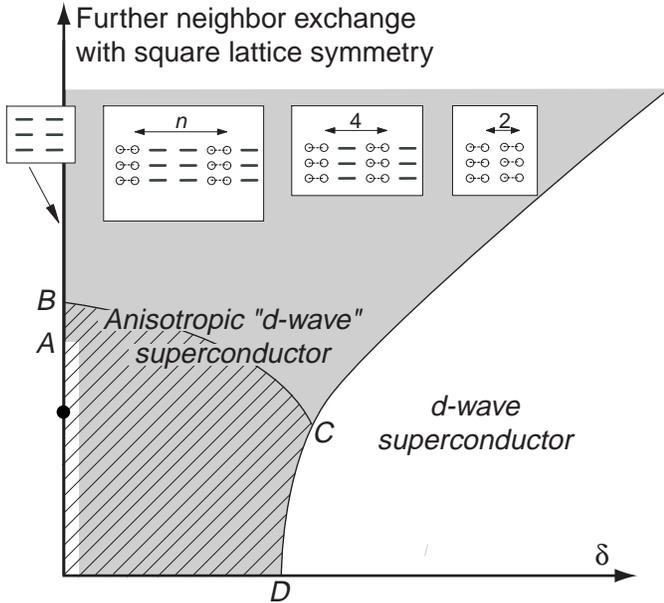


FIG. 1. Schematic $T = 0$ phase diagram (adapted from Ref. 9) for the high temperature superconductors as a function of a ratio of the near neighbor exchange interactions and the hole concentration, δ ; *e.g.* the vertical axis could be J_2/J_1 , the ratio of the first to second neighbor exchange. The shaded region has charge order *i.e.* at least one of the order parameters, $\phi_{x,y}$, in (2) is non-zero. The hatched region has broken spin-rotation symmetry with $\langle \vec{S} \rangle \neq 0$, and at least one of the order parameters $\Phi_{x,y\alpha}$ in (4) is non-zero; $\langle \vec{S} \rangle = 0$ elsewhere. The unfrustrated, insulating antiferromagnet with long-range Néel order is indicated by the filled circle. At $\delta = 0$, there is an onset of charge order above the point A, while spin-rotation invariance is restored above B (for a review of the $\delta = 0$ physics, see Ref. 6). The magnetic ordering quantum transition for $\delta > 0$ along the line BC is expected to be in the same universality class as that at the point B at $\delta = 0$, and is discussed as case (I) in Section III B. The magnetic ordering transition along the line CD differs slightly and is discussed as case (II) in Section III B. Neither of these theories apply to the multicritical point C, in the vicinity of which the transition is expected to be first order⁷. It is not known whether doping the unfrustrated antiferromagnet (denoted by the filled circle) leads to a system which crosses line BC or CD. The nature of the charge orders as determined by the computations of Ref. 8–11 are indicated at the top of the figure; numerous transitions, within the gray shaded region, in the nature of the charge ordering are not shown. The ground state at very low non-zero doping is an insulating Wigner crystal and there is subsequently an insulator-to-superconductor transition; superconductivity is present over the bulk of the $\delta > 0$ region. The central idea behind our approach is that many essential aspects of the spin excitation spectrum of the insulating, paramagnetic region ($\delta = 0$, $\langle \vec{S} \rangle = 0$) lead to a simple and natural description of the analogous properties of the *d*-wave superconductor.

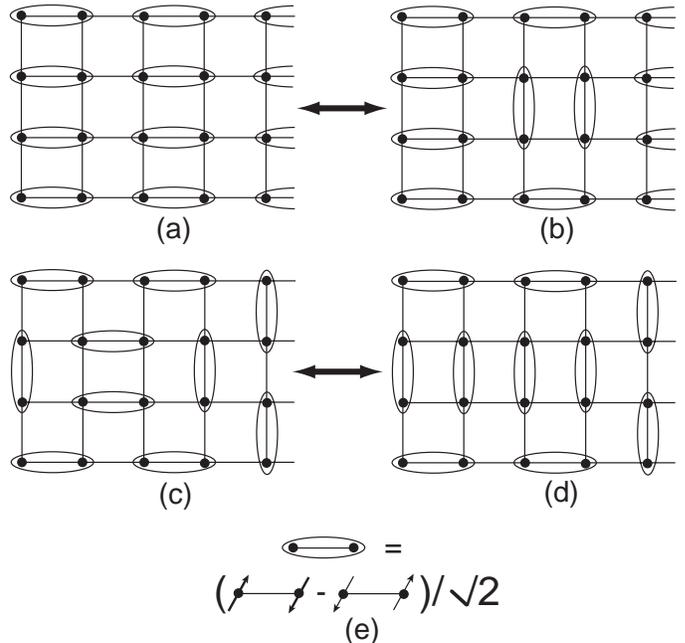


FIG. 2. Representation of antiferromagnetic spin correlations in the paramagnetic Mott insulator in the language of the quantum dimer model¹³. Each picture is a snapshot of nearest-neighbor singlet valence bonds, represented by ellipses as indicated in (e). The states (a) and (b) (and similarly, the states (c) and (d)) are connected to each other by resonance around a plaquette. The state in (a) has the maximal number of plaquettes around which such resonance can occur, and this lowers its energy. It has been argued⁶ that this effect leads to a broken lattice symmetry in two dimensions, and the resulting ground state has bond-centered charge order. A plausible candidate for such a state is the one with spin-Peierls order—this state breaks the square lattice symmetry in the same pattern as the state in (a). The presence of this broken symmetry will also modulate the bond charge density (*i.e.* the charge density on an orbital which is a linear combination of the orbitals on the two sites at the ends of the link) with period 2.

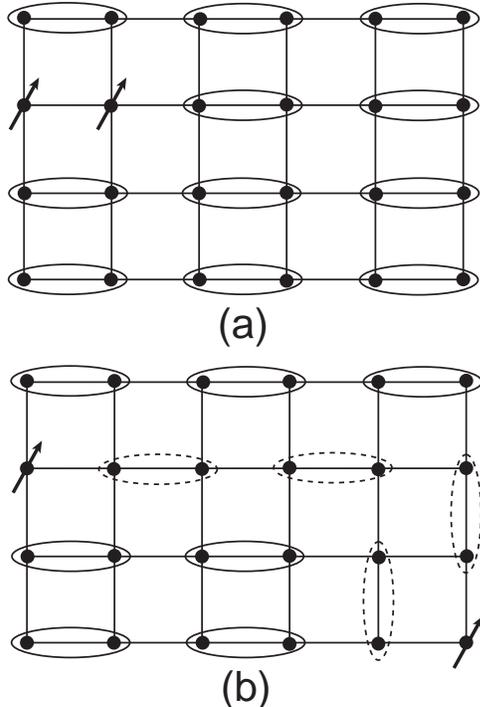


FIG. 3. (a) Cartoon picture of the bosonic $S = 1$ exciton of the insulating paramagnet—a similar picture is also expected to apply in the superconductor. (b) Fission of the $S = 1$ excitation into two $S = 1/2$ spinons. Notice that the spinons are connected by a “string” of valence bonds (denoted by dashed lines) which are not able to resonate with their environment: this string costs a certain energy per unit length and leads to the confinement of spinons.

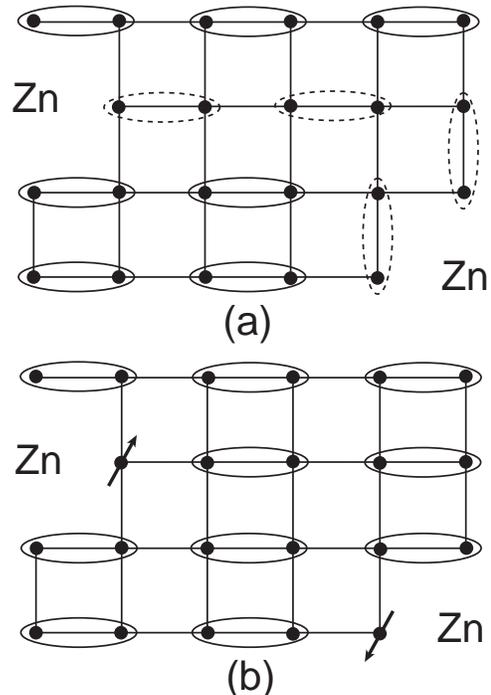


FIG. 4. Confinement of a $S = 1/2$ moment near a non-magnetic Zn impurity¹⁵. We consider two Zn impurities and imagine moving them infinitely far apart. In the trial state in (a), all Cu spins are paired into singlet bonds; while this is preferable when the Zn impurities are not too far, there is a string of valence bonds connecting them which are not able to resonate, and this will eventually lead to the preference of the structure in (b). In this case there is no string defect in the spin-Peierls order, but each impurity has a “free” moment in its vicinity. There is of course a very weak residual coupling between these moments even when they are far apart, and so the lowest energy states will be a singlet and triplet separated by a gap which decreases exponentially with the separation between the impurities. This should be contrasted with the case when moments do not form around each impurity (as may be expected in a deconfined paramagnet) when the gap above the ground state saturates at a fixed finite value as the impurities move far apart.

to the uniform magnetic susceptibility due to a single localized deformation (an “impurity”) obeys

$$\chi_{\text{imp}}(T \rightarrow 0) = \frac{\mathcal{C}}{k_B T} \quad (1)$$

with $\mathcal{C} = 1/4$. The form (1) is very familiar as the Curie high temperature limit in a large variety of systems. However, we are interested here in the opposite $T \rightarrow 0$ limit, and then (1) can arise only under special circumstances. (Note also that in any real experiment, there is a finite density of impurities, and even under the special circumstances (1) will only hold down to some very low temperature below which interactions between the impurities have to be accounted for.) We now consider a wide class of correlated electron ground states and list those that can obey (1) for an arbitrary localized impurity.

(i) *Fermi liquid*: A local spin degree of freedom will obey (1) only if its exchange interaction with the fermionic quasiparticles is ferromagnetic: then we must have $\mathcal{C} = S(S+1)/3$ with S integer or half-integer. If the exchange is antiferromagnetic (as is surely the case in the cuprates) then Kondo screening removes the moment as $T \rightarrow 0$ and χ_{imp} saturates at a finite value.

(ii) *Insulator with Néel order*: The magnetic order with $\langle \vec{S} \rangle = 0$ picks a preferred direction in spin space, and the impurity spin is not free to rotate. The magnetic susceptibility is finite only in the direction transverse to the Néel order, and the pinning of the spin rotation by the magnetic order leads to a finite χ_{imp} as $T \rightarrow 0$. So there is no $S = 1/2$ moment in a Néel state¹⁶.

(iii) *Spin gap insulator*: As we have shown in Fig 4, it is easy to find configurations (even with non-magnetic impurities) which have $\mathcal{C} = S(S+1)/3$ with S integer/half-integer.

(iv) *Magnetic quantum critical point*: We recently studied¹⁶ the critical point between (ii) and (iii) in two dimensions. We found there that (1) continues to be obeyed but \mathcal{C} is an irrational number—thus, remarkably, the impurity is associated with an irrational spin.

(v) *d-wave superconductor*: This case similar to the Fermi liquid in that it is possible to have Kondo screening of the moment as $T \rightarrow 0$. However the conditions for Kondo screening are much more stringent¹⁷ because of the lower density of fermionic states at low energy: it is only present if the antiferromagnetic exchange and the degree of particle-hole asymmetry exceed some finite value. In the free moment phase $\mathcal{C} = S(S+1)/3$ with S integer or half-integer, while at the impurity quantum critical point between the free moment and Kondo screened phases \mathcal{C} is an irrational number²⁰.

The NMR observations show that there is no Kondo screening of the moment at low doping, and that there is an onset of a non-zero Kondo temperature (T_K) at a critical doping beyond which the T_K rises rapidly. These results can be very naturally explained in a model of a $S = 1/2$ moment interacting with a d -wave superconductor, as has been shown in recent work^{18–20}. The separate

question of how the underdoped superconductor has a moment near a non-magnetic impurity in the first place is answered for “free” by the theoretical framework of Fig 1. Moving along the insulating vertical line at $\delta = 0$ in Fig 1, we proceed from case (ii) to case (iii) via (iv): a precise theory of the formation of the moment along this route has been presented¹⁶. Then we dope the paramagnetic charge-ordered insulator to reach the superconductor with $\langle \vec{S} \rangle = 0$; along this path the question is not one of the *formation* of the moment, but instead of its *quenching* by the fermionic quasiparticles; the latter issue is precisely that addressed by the Kondo model. Full translational symmetry must be restored at some *bulk* quantum critical point along this path (to be discussed in Section III), but this point will not, in general, coincide with the *impurity* quantum critical point at which T_K first becomes nonzero.

Next we turn to STM measurements of non-magnetic impurities²¹: these show a sharp low bias peak in the tunneling conductance at very low T in optimally doped samples. It was argued in Ref. 18 that this peak is related to a Kondo resonance in the d -wave superconductor and so its bias and width are determined by T_K . An immediate prediction is that there should be dramatic change in the bias dependence and spatial distribution of the tunneling conductance as the T is increased through T_K . In particular, the states near the impurity cross over from a coherent Kondo resonance to ones determined by potential scattering off the chemically distinct orbitals on the Zn site.

A more subtle issue, which is far from resolved, is that of the spatial dependence of the low-bias peak in the low T STM. In Ref. 18 the author and collaborators proposed that the unusual spatial dependence may be explained by Kondo scattering from a spatially diffuse $S = 1/2$ moment on the four nearest neighbor sites of the Zn moment. However, this computation did not account for the strong potential scattering from the central Zn site^{22,23}, and also the possible ‘blocking’ or ‘filter’ effect of the intervening BiO layer between the STM tip and the CuO₂ layer^{24–26}. The work of Zhu and Ting²³, and our discussions above and in Section III instead suggest the following model for future study: over the time scale of an electron tunneling event it is possible that local “stripe” correlations preferentially pick out a specific nearest neighbor site of the Zn impurity upon which the moment resides, just as in Fig 4. A Kondo model with the moment localized on this site, along with a strong repulsive potential on the Zn site, should then be solved for its local density of states. The actual STM measurements, of course, constitute a long time average of the tunneling current and this is obtained from the theory by an *incoherent* average of the results of the four possible positions of the local moment. Note that it is the relatively long-lived stripe order which demands an incoherent average, in contrast to the coherent average used in Ref. 18 which neglected stripe correlations.

III. SPIN AND CHARGE ORDERING QUANTUM TRANSITIONS

We will only consider spin and charge order in an idealized doped antiferromagnet, where any random disorder potential generated by the dopant ions is neglected. In the presence of disorder, the spin order (and concomitant charge order) may appear as spin glass order²⁷. Related ideas have been discussed by Zaanen²⁸.

A. Charge order

We begin by reviewing simple Landau-theory-like considerations to characterize charge order in doped antiferromagnets⁷. We consider charge density wave fluctuations in the (1,0) and (0,1) directions for a lattice in which the Cu ions occupy the sites at integer values of x , and the O ions occupy half-integer x . Then we can describe the spacetime ($r = (x, y)$) modulations in the charge density, $\delta\rho(r, \tau)$ by

$$\delta\rho(r, \tau) = e^{iK_c x} \phi_x(r, \tau) + \text{c.c.} + x \rightarrow y \quad (2)$$

where K_c is the ordering wavevector and $\phi_{x,y}$ are *complex* order parameters. For commensurate $K_c = 2\pi m/n$ (m, n , integers) the coupling to the underlying lattice will allow a term in the action

$$\lambda_c \int d^2 r d\tau \phi_x^n(r, \tau) + \text{c.c.} + x \rightarrow y. \quad (3)$$

This term pins the phase of $\phi_{x,y}$ either at $\arg[\phi_{x,y}] = 0$ for $\lambda_c < 0$, or at $\arg[\phi_{x,y}] = \pi/n$, for $\lambda_c > 0$ (changes of $\arg[\phi_{x,y}]$ by integer multiples of $2\pi/n$ are not significant as they correspond simply to a translation in (2) by integer lattice spacing); the first case corresponds to a *site-centered* charge density wave, while the second is *bond-centered*. The state in Fig 2a has $m = 1, n = 2$ and is bond-centered; note that for this case only the charge density on all the Cu sites in unmodified—the modulation is apparent if we examine the charge density on the O sites, or in off-site correlations of the Cu orbitals which take the form of a p_x density-wave state²⁹.

To choose between the different possibilities for (2) we have to turn to more microscopic considerations. In the region with $\langle \vec{S} \rangle = 0$ in Fig 1, a controlled large N theory^{8,9} was used to follow the evolution of the $\delta = 0$ bond-charge order in Fig 2a to non-zero δ : we found only bond-centered waves with n *even*. The value of n jumped to a large number at small δ and then decreased monotonically through a series of even integer plateaus with increasing δ (see the top of Fig 1); in particular there was large plateaus at $n = 4$ (and occasionally a small plateau at $n = 2$) before translational symmetry was eventually restored to an isotropic d -wave superconductor. (Specific theories of hole doping and fermionic excitations in states with $n = 2$ charge order¹¹, and also with “plaquette” order³⁰, have been presented recently.) We have

proposed that such bond-centered charge order with even n is characteristic of the high temperature superconductors in the region with no spin order *i.e.* $\langle \vec{S} \rangle = 0$; in the regime where the ground state is an isotropic d -wave superconductor (see Fig 1) strong low energy fluctuations of such bond charge order can be expected. Some experimental support for this picture has emerged in recent measurements of the phonon spectra^{31,11}.

The nature of the quantum theory describing the eventual restoration of translational invariance has also been discussed⁹: the critical fixed point between a d -wave superconductor and a d -wave superconductor with co-existing charge order is described by a relativistic field theory with dynamic exponent $z = 1$ which obeys hyperscaling and exhibits ω/T scaling. This should be contrasted with the work of the Rome group³² who have considered charge ordering transitions in Fermi liquids: such fixed points are very different and do not satisfy any of the characteristics just mentioned.

B. Spin order

A related analysis can be developed for spin excitations in the doped Mott insulator. Here it is convenient to begin in the paramagnetic Mott insulator at $\delta = 0$ which, as shown in Fig 3, has a sharp $S = 1$ exciton above a spin gap. Upon moving to the doped superconductor, it is easy to see that this exciton retains its integrity for a finite range of δ . The superconductor does have an entirely new class of low energy spin excitations which are not present in the insulator: these are the fermionic $S = 1/2$ Bogoliubov quasiparticles. However, these low energy quasiparticles reside only in certain sectors of the Brillouin zone and momentum conservation constraints can prevent the $S = 1$ exciton from decaying into two $S = 1/2$ quasiparticles. Explicit calculations exhibiting this feature appear in Ref. 11. So the main effect of the non-zero δ will be renormalize the dispersion of the exciton: in particular, it is likely that the exciton will eventually move its minimum away from (π, π) to a general wavevector $(\pi, \pi) + K_s$. As in the discussion of the charge density, we assume that K_s is polarized along (1,0) or (0,1). In the presence of such excitonic fluctuations, the spin density on the Cu sites is given by:

$$S_\alpha(r, \tau) = (-1)^{x+y} e^{iK_s x} \Phi_{x\alpha}(r, \tau) + \text{c.c.} + x \rightarrow y \quad (4)$$

where $\alpha = x, y, z$ are the spin directions, and $\Phi_{x\alpha}, \Phi_{y\alpha}$ are complex field operators for the $S = 1$ excitons. We now discuss various terms in the effective action for $\Phi_{x\alpha}, \Phi_{y\alpha}$. For the reasons just discussed, we will neglect their couplings to the fermionic quasiparticles, but it is not difficult to include such a coupling if momentum conservation happens to permit it^{9,33}. First, along the lines of Refs. 9,16,34 we have the usual gradient and time derivative terms present in any quantum antiferromagnet:

$$\begin{aligned}
\mathcal{S}_\Phi = \int d^2r d\tau & \left[|\partial_\tau \Phi_{x\alpha}|^2 + c_1^2 |\partial_x \Phi_{x\alpha}|^2 + c_2^2 |\partial_y \Phi_{x\alpha}|^2 \right. \\
& + |\partial_\tau \Phi_{y\alpha}|^2 + c_1^2 |\partial_y \Phi_{y\alpha}|^2 + c_2^2 |\partial_x \Phi_{y\alpha}|^2 + s(|\Phi_{x\alpha}|^2 + |\Phi_{y\alpha}|^2) \\
& + \frac{u_1}{2} \{(|\Phi_{x\alpha}|^2)^2 + (|\Phi_{y\alpha}|^2)^2\} + \frac{u_2}{2} \{|\Phi_{x\alpha}^2|^2 + |\Phi_{y\alpha}^2|^2\} \\
& \left. + v_1 |\Phi_{x\alpha}|^2 |\Phi_{y\beta}|^2 + v_2 |\Phi_{x\alpha} \Phi_{y\alpha}|^2 + v_3 |\Phi_{x\alpha}^* \Phi_{y\alpha}|^2 \right]. \quad (5)
\end{aligned}$$

Second, the underlying lattice may pin K_s to the commensurate values $K_s = 0$ or $K_s = 2\pi/(2p)$ with p integer. For the special case $K_s = 0$, we see from (4) that $\Phi_{x,y}$ have the same spin modulation and so only one of them need be considered; further they should also be real, and so \mathcal{S}_Φ reduces to the familiar $O(3)$ φ^4 field theory in 2+1 dimensions³⁴. For $K_s = \pi/p$, as in (3), we have a term in the action

$$\mathcal{S}_L = \lambda_s \int d^2r d\tau (\Phi_{x\alpha}^2)^p + c.c. + x \rightarrow y \quad (6)$$

which again pins the phase of $\Phi_{x\alpha}$ and $\Phi_{y\alpha}$ to a discrete set of allowed values. Finally, as noted by Zachar *et al.*⁷ there is also a coupling between the spin and charge degrees of freedom

$$\mathcal{S}_{\Phi\phi} = w \int d^2r d\tau \phi_x^* \Phi_{x\alpha}^2 + c.c. + x \rightarrow y \quad (7)$$

which is operative only if $K_c = 2K_s$ and then pins the long-range spin and charge order to each other. The ordering observed in LNSCO is of this type with $m = 1$, $n = 4$, and $p = 4$. Spin correlations pinned at $p = 4$ are also universally observed over a wide doping range in the cuprates: this pinning is difficult to understand from the semiclassical theory of stripe formation³⁵, and we propose that it is linked via (7) to singlet bond charge order correlations for which a large plateau at $n = 4$ was found in Ref. 9.

The above framework can now be used to study the transition into the magnetically ordered region with $\langle \vec{S} \rangle \neq 0$ in Fig 1. This transition is driven by decreasing the value of s in (5) and corresponds to a condensation of the $S = 1$ exciton. The nature of this transition depends crucially on the role of the charge order, and two distinct cases can be separated.

(I) In the first case, long-range charge order remains an innocent spectator to the spin ordering transition. If the spin ordering is at a wavevector $K_s = K_c/2$, then the charge ordering, via the coupling (7), will select $\Phi_{x\alpha}$ over $\Phi_{y\alpha}$ (say), and also pin its phase so that $\Phi_{x\alpha}$ is real. The theory reduces to that of a single real 3-component vector, and is again the familiar $O(3)$ φ^4 field theory³⁴. This situation is the case along the BC portion of the spin-ordering phase boundary in Fig 1.

(II) In the second case, the spin-ordering fluctuations are paramount, and charge order appears only in the region with $\langle \vec{S} \rangle \neq 0$ (assuming the spin correlations are not spiral); this is the case along the CD portion of the phase

boundary in Fig 1. The critical theory is now given by $\mathcal{S}_\Phi + \mathcal{S}_L$ and the spectator charge order is determined simply by (7) as $\phi_x \sim \Phi_{x\alpha}^2$, and similarly for ϕ_y .

Finally, we note the nature of the dynamic spin fluctuation spectrum near the spin-ordering transition in the region with $\langle \vec{S} \rangle = 0$. Here, an adequate picture emerges if we simply treat (5) in the Gaussian approximation, with $s > 0$. A combination of (5) and (4) then implies the dynamic spin susceptibility³³:

$$\begin{aligned}
\chi(k, \omega) \propto & \frac{1}{c_1^2(k_x - G_{+x}^x)^2 + c_2^2(k_y - G_{+y}^x)^2 + s - (\omega + i\eta)^2} \\
& + \frac{1}{c_1^2(k_x - G_{-x}^x)^2 + c_2^2(k_y - G_{-y}^x)^2 + s - (\omega + i\eta)^2} \\
& + \frac{1}{c_1^2(k_y - G_{+y}^y)^2 + c_2^2(k_x - G_{+x}^y)^2 + s - (\omega + i\eta)^2} \\
& + \frac{1}{c_1^2(k_y - G_{-y}^y)^2 + c_2^2(k_x - G_{-x}^y)^2 + s - (\omega + i\eta)^2} \quad (8)
\end{aligned}$$

where $G_{\pm}^x = (\pi \pm K_s, \pi)$ and $G_{\pm}^y = (\pi, \pi \pm K_s)$, and η is a positive infinitesimal. A related spectrum has been discussed recently by Batista *et al.*³⁶ but on the ordered side of the transition: they have argued that the superposition of the terms in (8) leads to a maximum in the spectral density at the commensurate point $k = (\pi, \pi)$ and this is responsible for the ‘‘resonance peak’’ in neutron scattering.

IV. EFFECT OF AN APPLIED MAGNETIC FIELD

In this final section we briefly review recent work³⁷ on the effect of an applied magnetic field, oriented perpendicular to the CuO_2 layers, on the above spin and charge ordering transitions within a superconducting ground state.

So far, we have found that there was little material difference between the spin/charge ordering transitions of a superconductor and those of an insulator: the superconductor has additional low energy quasiparticle excitations, but these often decouple from the critical spin/charge degrees of freedom because of constraints from momentum conservation. The situation changes dramatically in the presence of an external field. Briefly stated, the ‘‘background’’ superconducting order has an infinite diamagnetic susceptibility and this feeds into an anomalously strong response of the spin/charge order, much stronger than that would be found for the corresponding quantum transitions in an insulator. The effect is quite simple: in the presence of an applied field, H , the type II superconductor reacts by setting up superflow currents which screen the field. A standard calculation using the standard Ginzburg Landau free energy shows that

mean kinetic energy density of superflow $\sim \left| \frac{H}{H_{c2}} \right| \ln \left| \frac{H_{c2}}{H} \right|$.

(9)

This shift leads to concomitant change in the effective action for the spin/charge order³⁷ via the simple Landau theory coupling (emphasized in Ref. 38)

$$\mathcal{S}_{sc} = \kappa \int d^2r d\tau |\psi|^2 |\Phi_{x\alpha}|^2 + x \rightarrow y \quad (10)$$

(and similarly for $\phi_{x,y}$) to the superconducting order parameter ψ . Crudely speaking, this correction is accounted for an H -dependent shift in the value of s in (5) which has the form³⁷

$$s(H) = s - \tilde{\kappa} \left| \frac{H}{H_{c2}} \right| \ln \left| \frac{H_{c2}}{H} \right|. \quad (11)$$

This simple expression now has a number of strong consequences for the position of the phase boundary for the onset of long-range spin order, and for the nature of the spin excitation spectrum on either side of the phase boundary. On the side with long-range spin order, (11) implies that the Bragg elastic scattering intensity should increase (for $\tilde{\kappa} > 0$) as³⁷ $\sim (|H|/H_{c2}) \ln(H_{c2}/|H|)$: this prediction is consistent with recent experiments³⁹. On the side with no spin order, the dynamic susceptibility will be given by (8) but with s replaced by $s(H)$: hence the minimum excitonic energy, $\sqrt{s(H)}$, will decrease with increasing H —this trend is also consistent with experiments⁴⁰.

All of the above effects considered the uniform, spatially averaged, consequence of the applied field. However, as is well known, the superflow kinetic energy in (9) is inhomogeneously distributed in the form of a vortex lattice, and this has interesting observable consequences for the spin/charge order. In the spin-ordered phase, there are weak satellite Bragg peaks^{41,37} around $G_{\pm}^{x,y}$, separated from $G_{\pm}^{x,y}$ by reciprocal lattice vectors of the vortex lattice. In the region without spin order ($\langle \vec{S} \rangle = 0$) which was considered in Ref. 37, the wavefunction, $\Phi_{x,y\alpha}(r)$, of the lowest energy exciton (which is an excited state at energy $\sqrt{s(H)}$) will be peaked at the vortex core, but will extend will outside the vortex core to a distance $\sim c_{1,2}/\sqrt{s(H)}$ (up to the natural maximum of the vortex lattice spacing), where the velocities $c_{1,2}$ are expected to be of order the spin-wave velocity in the antiferromagnetic state at $\delta = 0$. Given the large size of the exciton wavefunction, small local disorder may well enhance $\Phi_{x\alpha}$ over $\Phi_{y\alpha}$ (or vice versa) in a given set of vortices. Associated with this excitonic excited state is a spin modulation specified by $\Phi_{x,y\alpha}(r)$ and (4), and a charge modulation given by $\phi_{x,y}(r) \sim \Phi_{x,y\alpha}^2(r)$ and (2).

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