ORDER AND FRUSTRATION ON A RANDOM TOPOGRAPHY

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Applications of Field Theory to Statistical Mechanics, Proceedings, Sitges Spain 1984, edited by L. Garrido, Springer-Verlag, Berlin (1985)

1. INTRODUCTION

A study of frustrated systems of particles in two dimensions is of interest because they serve as a model for amorphous states of matter and are analytically tractable.[1,2,3] By "frustration" we mean that the particles in the ground state cannot simultaneously sit in the minima presented to them by pairwise interactions with their neighbors. We can introduce frustration into two dimensional particle configurations by imbedding them in a randomly corrugated surface.[4] A microscopic argument by Gaspard et al.[4] shows that the density of disclinations, i.e., particles with other than 6 nearest neighbors, is directly related to the gaussian curvature $K(\vec{r})$ of the surface,

$$\sum_{Z} (6-Z)N_Z = \frac{3}{\pi} \int\int_S \sqrt{g}\; d^2r\; K(r) . \qquad (1.1)$$

Direct experimental realizations of such systems may be possible, by quenching a viscous liquid like molten quartz below its glass transition and then absorbing particles at the interface. For the theory presented in this paper to be applicable we require that the wavelength of the quenched fluctuations be much longer than the interparticle spacing, and that the forces confining the particles to the interface be normal to the surface.

We describe the displacement of the corrugated surface from a reference flat surface by a quenched random field $f(x,y)$.

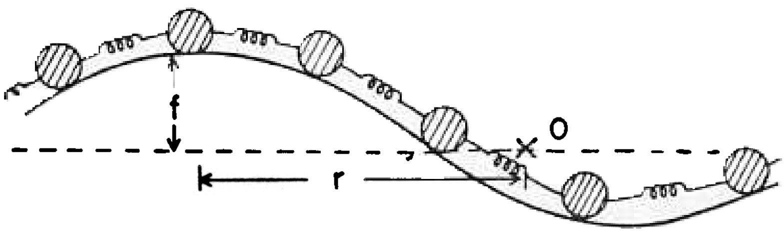

Fig. 1. Array of Atoms on a Random Topography

We will consider two models for the probability distribution of the shape of the surface: A "rough surface" [5] which has a distribution

$$P_I\ \{f(\vec{r})\}\ \propto\ \exp\Big\{-\frac{1}{2T_f}\ \ \vec{\nabla}f\big|^2\,\frac{d^2r}{2} \qquad (1.2)$$

and a "smooth surface" with

$$_{II}\ \{f(\vec{r})\ \propto\ \exp\Big\{-\frac{\ }{2T_f'}\ \left(\frac{f}{a}\right)^2\ \frac{d'}{2} \qquad (1.3)$$

Our results will be insensitive to the particular kind of surface chosen.

We use a long wavelength continuum elastic theory to describe the energetics of the particles on the surface and a synopsis of our results is as follows: The random topography induces local regions of positive and negative gaussian curvatures with a probability distribution that is given by

$$P\ \{K(q)\qquad \exp\Big\{-\frac{\alpha}{2T_f^{\,2}}\sum_q\ \frac{\tilde{K}(q)\tilde{K}(-q)}{q^4} \qquad 1.4)$$

where K(q) is the Fourier transform of the local gaussian curvature and α is a number whose precise value depends upon the kind of surface that is under consideration. Despite the presence of this curvature, for small enough T_f, there is a finite band of temperatures in which the crystalline phase is stable; the only effect of the frustration being a renormalization of the long wavelength elastic constants. More, importantly, however, at low temperatures when the screening by the dislocation pairs become sluggish, a prolification of unbound dislocations (i.e., disclinations dipoles) attempts to compensate the frozen distribution K(r). This leads to a reentrant melting transition into the hexatic phase. The resulting hexatic phase is also unstable at low enough temperatures, because the core energy of the disclinations is driven negative by the frustration, leading to a further reentrant transition. The low temperature phase with lack of long range order might exhibit glassy behavior in its transport coefficients. The possible phase diagrams which emerge from these considerations are as follows:

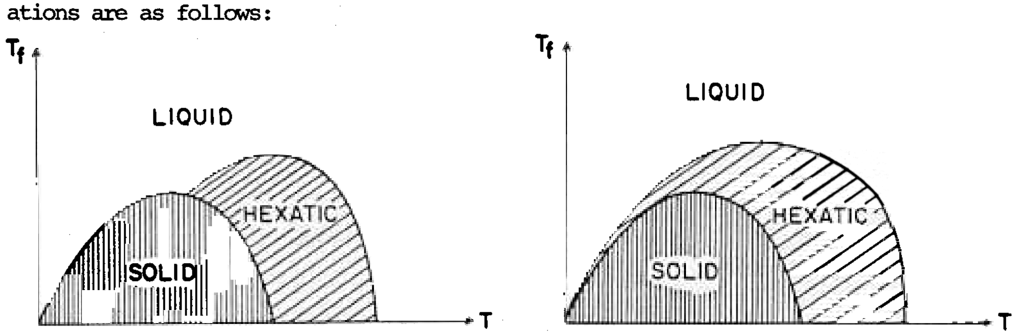

Fig. 2. Two possible phase diagrams of the system.

2. DESCRIPTION OF THE CALCULATION

A complete description of the calculation is in Ref. 6; here we present a short summary. The displacement of the particles from their equilibrium positions in flat space is described by a strain field $u_{ij}(\vec{r})$. In the ground state the system will attempt to compensate the underlying corrugation by introducing a strain field $u^{o}_{ij}(r)$. The disclination density contained in this strain field can be shown to be equal to $K(\vec{r})\ \sec\theta(\vec{r})$ where $\theta(\vec{r})$ is the angle between the normal to the corrugated surface and the normal to the reference plane. This relationship is exact for a radially symmetric surface but true only to second order in f for a general surface. Assuming that the continuum elastic free energy can only be a function of the local change in distances between the particles, we obtain the free energy

$$F\{e\} = \frac{1}{2}\int \frac{d^2r}{a^2}\ e_{ij}\, C_{ijk\ell}\, e_{k\ell} \qquad (2.1)$$

with

$$e_{ij} = u_{ij} + \frac{1}{2}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}$$

where $C_{ijk\ell}$ is an elastic tensor. If we ignore the presence of defects (this will be shown to be valid at long wavelengths for certain ranges of temperature and T_f) we find that the only effect on the phonon degrees of freedom is a renormalization of the elastic constants.

We now include defects in our theory. If there are disclination charges s_α at positions $\vec{r}_\alpha$ and dislocation charges $\vec{b}_\beta$ at positions $\vec{r}_\beta$ then the total disclination density is

$$s(\vec{r}) = \sum_\alpha \delta(\vec{r}-\vec{r}_\alpha)\, s_\alpha + a_o \varepsilon_{ij} \sum_\beta b^\beta_j\, \partial_i\, \delta(\vec{r}-\vec{r}_\beta) \qquad (2.2)$$

Inserting this defect density into the free energy we obtain the following expression describing the interaction of the defects among each other and with the background quenched random curvature

$$F_d = \frac{K_o}{2}\sum_{\vec{k}} [s(-\vec{k}) - K(-\vec{k})]\ \frac{1}{k^4}\ [s(\vec{k}) - K(\vec{k})] + E_c \sum_\alpha s^2_\alpha + E_p \sum_\beta |b_\beta|^2 \qquad (2.3)$$

where K_o depends upon the elastic constants and E_c and E_p are phenomenological core energies. We consider the possibility that free dislocations occur in large enough densities at long wavelengths to destroy long range crystalline order. Using the replica method the following renormalization group flows can be generated for K_o^{-1} and $\ell n y = -E_\alpha$ when $T_f < T_f^c$.

There is a fixed line at $y = 0$. The fixed line is unstable to the formation of dislocations at low temperatures, signalling the presence of a reentrant melting transition. For a range of temperatures between K_L^{-1} and K_+^{-1}, the crystalline phase is stable. For $T_f > T_f^c$ the crystalline phase is always unstable to a dislocation unbinding transition.

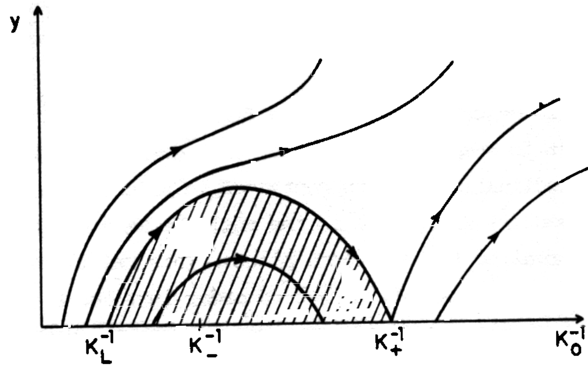

Fig. 4. Renormalization group flows for the dislocation unbinding transition.

There is a further instability in the resulting hexatic phase. The presence of a screening cloud of dislocations and the quenched random curvature leads to a decrease in the core energy of the disclinations.

$$E_c^{eff} = E_c + \frac{\xi_T^2}{a_o^2} (\gamma_1 E_p - \gamma_2 E_p^2 T_f^2) \tag{2.4}$$

where γ_1 and γ_2 are numerical constants and ξ_T is the translational correlation length The core energy is driven negative at low enough temperatures leading to another reentrant transition to a phase with no long range order.

It is a pleasure to thank E. Brézin, B. Halperin and J.F. Sadoc. This work was supported by the National Science Foundation through the Harvard Material Science Lab and through grant No. DMR-82-07431.

REFERENCE

1. D.R. Nelson, Phys. Rev. B27, 5515 (1983).
2. D.R. Nelson and B.I. Halperin, Phys. Rev. B19, 2457 (1979).
3. M. Rubinstein, B. Shraiman and D.R. Nelson, Phys. Rev. B27, 1800 (1982).
4. J.P. Gaspard, R. Mosseri and J.F. Sadoc, Proceedings of the Conference on the Structure of Non-Crystalline Materials, Cambridge, England (1982).
5. J.A. Weeks in Ordering in Strongly Fluctuating Condensed Matter Systems. ed. T. Riste (Plenum, New York 1980).
6. S. Sachdev and D.R. Nelson, J. Phys. C, in press.